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ALGORITHM FOR FIRE CONTROL

Technology Staff
Military Airplane Systems Division

D162-10026-1

June 1969

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THE BOEING COMPANY
Seattle, Washington

ALGORITHM FOR FIRE CONTROL

Work Performed in Partial Fulfillment
of Naval Weapons Center

Contract N00123-69-C-1344

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SUMMARY

This document describes a method of accurately computing, in airborne fire control computers, the impact range of free falling projectiles such as bombs or bullets. The document has two parts. In Part I older computation methods and their limitations are described briefly and compared with a new computation method, or algorithm. Part II presents a rigorous mathematical development of the algorithm with examples of its accuracy for several typical Navy weapons.

The new algorithm can contribute significantly to the flexibility and effectiveness of Navy strike aircraft. This method of calculation is applied through software, and it can be used in new systems or reprogrammed into existing systems without increasing hardware cost.

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ABBREVIATIONS AND SYMBOLS

a	Aircraft acceleration
A	Geometric altitude above sea level
A_{max}	Upper limit of geometric altitude in each atmospheric region
A_{min}	Lower limit of geometric altitude in each atmospheric region
AC	Coefficients of Chebyshev polynomials
A_T	Rocket acceleration due to thrust
C_D	Coefficient of drag of projectile
C_N	Coefficients in polynomial expansion of atmospheric model
D	Diameter of projectile
DC	Coefficients of Legendre polynomials
EC	Coefficients used in the Legendre expansion
F	General function
F_T	Thrust of rocket engine
g	Acceleration due to gravity
g_o	Local value of gravity at sea level
h	General integration interval
H	Drag function
H_A	Aerodynamic drag function
H_g	Geopotential altitude
H_R	Rocket thrust function
i	Number of integration steps taken so far
K_D	Coefficient of drag of projectile
LC	Coefficients used in the Legendre expansion
m	Polynomial index
M	Mass of projectile
M_R	Mass of rocket
M_Y	Number of steps to be taken with altitude (Y) as the independent variable
n	Polynomial index
N	Exponent in power series approximation of the atmospheric model
N_X	Number of integration steps to be taken with downrange (X) as the independent variable
r	Turn radius
P_i	The i^{th} Legendre polynomials

ABBREVIATIONS AND SYMBOLS (CONTD)

R_E	Local effective value of Earth's radius
S	Symbol used to represent the expression $2/(A_{max} - A_{min})$
t	Time – measured from release
t_D	Time of drogue deployment on retarded weapons
t_o	Time of release
t_R	Time variable used with rocket mass and rocket thrust functions
U_i	The i^{th} Chebyschev polynomial
V	Velocity
V_0	Velocity at release
V_X	Downrange component of velocity
V_{XD}	Downrange component of velocity at drogue deployment
V_{X0}	Downrange component of velocity at release
V_Y	Vertical component of velocity
V_{YD}	Vertical component of velocity at drogue deployment
V_{Y0}	Vertical component of velocity at release
x	General independent variable
X	Downrange position
X_D	Downrange position at time of drogue deployment
X_0	Downrange position at release
X_R	Impact range of projectile
Y	Altitude above sea level
Y_D	Altitude at drogue deployment
Y_0	Altitude at release
Δd	Distance traveled in one calculation interval (Δt)
$\Delta\theta$	Dive angle deviation in one calculation interval (Δt)
Δt	Time interval between successive impact range calculations
ΔV	Velocity deviation in one calculation interval (Δt)
ΔY	Altitude deviation in one calculation interval (Δt)
θ	Dive Angle
•	Dot above symbol denotes derivative with respect to time; n dots denotes n-th derivative
ρ	Air density (mass)
ρw	Air density (weight)

DEFINITIONS

Algorithm	A precisely defined, step-by-step method of computing some quantity.
Ballistic Wind	A constant, nonlayered wind which is equivalent to the actual wind in the respect that it affects the impact point by the same amount.
CBU	Clustered Bomb Unit–a canister-type bomb which dispenses smaller bombs at some point in its trajectory.
CEP	Circular Error Probable–the radius of a circle, centered about the expected impact point, into which a projectile will fall with a 50% probability.
Crossrange	The horizontal distance measured in a direction 90 degrees clockwise from the downrange direction.
Dispenser Weapon	Synonymous to CBU.
Downrange	The horizontal distance measured in a direction away from the point of projectile release and in the vertical plane which includes the release velocity.
Drogue	A parachute, vanes, or other device deployed at the rear of a bomb to increase its drag and slow it down.
Ejection	Frequently bombs are physically ejected from an aircraft to ensure clean separation. The velocity imparted to a bullet can be considered an ejection velocity.
Impact Range	The horizontal distance between the release and impact of a projectile.
RSS	Root Sum Square–the square root of the sum of the squares; a method used to add probabilistic effects of independent error sources.
Runge-Kutta	The name of a class of formulas used to evaluate integrals. They are widely used in calculating solutions of differential equations for which analytical solutions cannot be found.
Trajectory	The path of a projectile.
Word	The quantity of a computer memory normally required to store a single number or instruction.

PART I

GENERAL DESCRIPTION

1.0 INTRODUCTION

1.1 The Weapon Delivery Problem

Most aircraft weapons can be described as ballistic projectiles. That is, the only forces acting on them in flight are gravity and aerodynamic drag. Bullets, streamlined bombs, drogued bombs, clustered munitions, and unguided rockets (after burnout) are all ballistic projectiles. Guided weapons and weapons developing lift are not ballistic projectiles.

For successful weapon delivery, the pilot of an airplane must maneuver his plane into a position from which the weapon will fall to the target. The problem is made difficult in two ways. First, the release must be done accurately because the impact range is very sensitive to release errors. Figure 1 shows how the distance a bomb travels changes as bomb release conditions change. Heading, dive angle, speed, altitude, and range from the target must all be exactly right when the weapon is released. The second difficulty is selecting a correct combination of all these factors. The path of a ballistic projectile is curved; and the precise curvature depends on the atmospheric density, the size and shape of the projectile, and the speed and dive angle at release.

Several pilot aids and delivery tactics have been developed to ease weapon delivery problems. Bombsights, release timers, and computers are some of the devices; dive bombing is a delivery tactic which minimizes the effect of errors at weapon release.

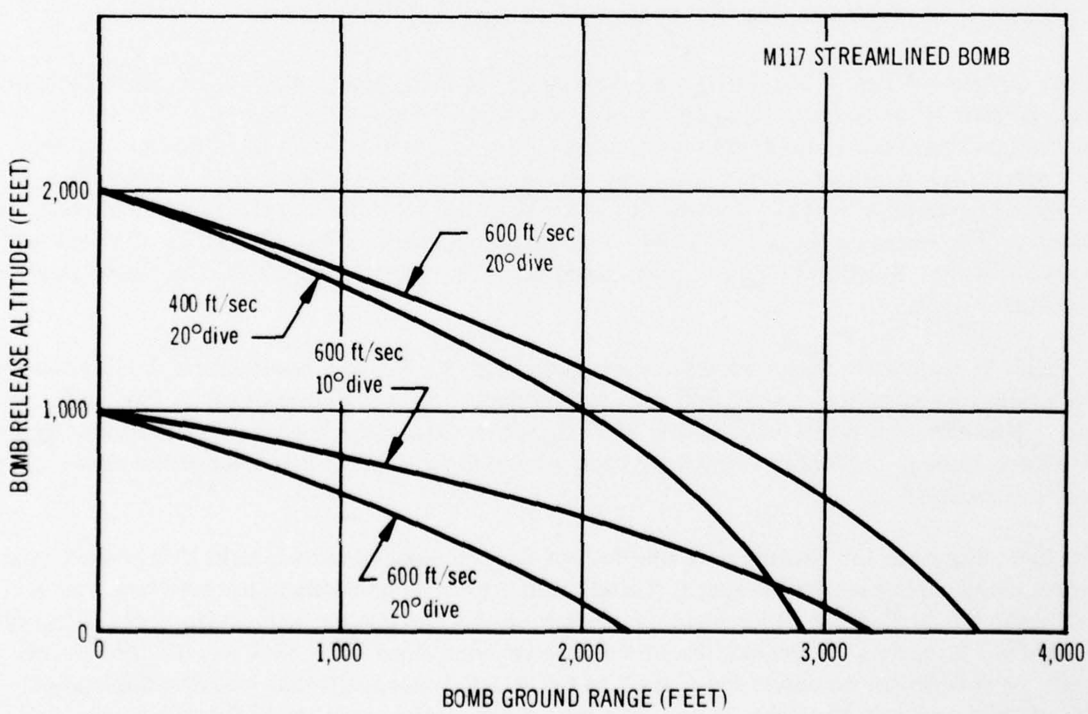

Figure 1. Bomb Trajectories

The obvious solution to this problem is to have the fire control system calculate the correct range to release the weapons for any speed, altitude, and dive angle. Because this calculation is so difficult, no operational fire control system can accurately compute range for all the weapons and delivery conditions used. The inability to perform this calculation accurately is a significant source of weapon delivery error for all current aircraft. Equally important, delivery tactics become constrained to those for which the fire control system can do a fairly good job.

The subject of this document is a new computing method, or algorithm, for the calculation of weapon range. This method can be used for any ballistic projectile, and it produces accurate answers for any release condition operationally possible. This algorithm can be used in current or future fire control systems having a digital computer.

1.2 History of Calculation Methods

The path of a ballistic projectile can be described by a relatively simple-appearing set of differential equations. If drag is left out, numerous complications can be added to the basic equations of motion (e.g., the rotation of the earth), and a mathematical solution can still be found. This is not true once drag has been included, because an accurate description of the drag is mathematically difficult. Drag is a function of the projectile's size, speed, and drag coefficient, as well as air density. Air density varies primarily with altitude and secondarily with conditions such as the weather, latitude and time of year. The drag coefficient varies with Mach number, which in turn varies with speed and air temperature, which varies with altitude and so on, just as atmospheric density does. These factors are empirically determined and are nonlinear. At any given speed or altitude the drag acting on a projectile can be calculated, but an accurate function describing the drag cannot be put in the differential equations to solve for the time of fall or point of impact.

A way around the lack of a solution has been known for a long time. All the bombing tables and other ballistic tables are calculated by a process known as "numerical integration." In this process, the total path is divided into many pieces, and each piece is worked with an approximation that all forces remain constant for short time periods. A mathematical formula with constant forces permits easy calculation of the position and velocity of the projectile at the end of 0.1 second, for example. The forces on the projectile are calculated for the new point and used for the next 0.1 second. Figure 2 illustrates how this process can be made as accurate as desired by taking smaller and smaller steps.

A typical ballistic table trajectory calculation may divide the fall of a bomb into a thousand segments. This yields very accurate answers, and each answer takes only a few seconds on an IBM 7094 computer. However, this is too long and the 7094 computer is too large for aircraft fire control systems. Therefore, numerous other methods have been used in the aircraft to provide weapon range information to the pilot.

The first advance in fire control computation was the manually adjustable sight ("depressed reticle"). Before flight, a pilot selects the speed, altitude, and dive angle he wants to use to attack a target. He goes to a ballistic table which gives the angle by which the weapon will deviate from straight line travel when it reaches the ground, if released at those conditions. The pilot sets his sight to this angle. Now if he can maneuver the aircraft to the selected speed, altitude, and dive angle at the instant the target is in the sight, he will get a hit. A deviation in any one of them, however, will result in a large miss. Accurate weapon delivery can be done with such a system, but it requires excellent pilot control. This degree of control can be attained only by using intensively practiced

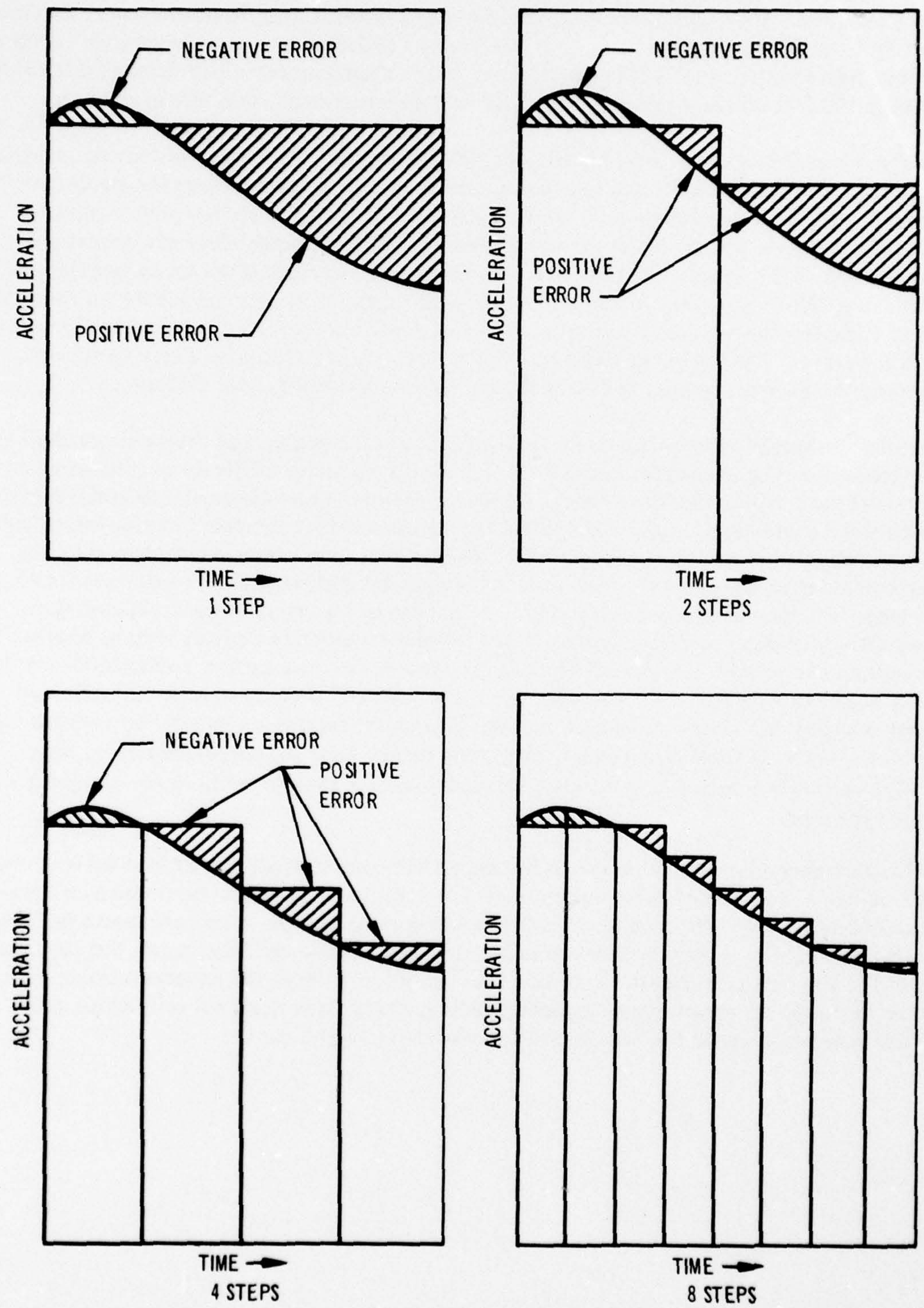

Figure 2. Numerical Integration with Different Numbers of Steps

delivery maneuvers. Always using the same maneuver in combat is dangerous; moreover, the finesse achieved by a test pilot on a practice range is not readily duplicated by a green pilot under combat conditions. As a result, aircraft with relatively simple fire control systems may demonstrate delivery errors under 100 feet on a test range, but produce average errors over 1,000 feet in combat.

The most modern systems (e.g., F-111, A-6A) provide "universal" mode weapon delivery, using a digital computer in the fire control system continuously compute weapon range for current conditions (speed, altitude, dive angle, wind). Now displays can be driven to aid the pilot in steering the aircraft to a release point. In one example, a heads-up display shows where the impact point of a weapon would be if released right now, and how that point will move if the speed, heading, etc., are maintained. Since the pilot can also see the target through the display, he has the information needed to maneuver the airplane smoothly to a release point, and he maintains considerable latitude in just how to do it. This degree of flexibility and naturalness contributes to a very significant improvement in the operational usability of the aircraft and the accuracy of delivery.

The modern "universal" systems are not sufficiently accurate, however, and their universality has some severe limits. The inaccuracy is due to the previously discussed difficulty of calculating a ballistic trajectory. Approximations have been made to keep the calculation simple and fast enough for the airborne computer. Usually a simplified form is assumed for the drag function which permits the differential equations of motion to be solved analytically. This solution is evaluated by the airborne computer. For a given type weapon, this simplified approximation can work well over a limited range of release conditions. Outside these conditions, the errors in the approximation gradually introduce larger and larger errors. For a different weapon or delivery regime, another approximation and set of equations are needed. As a result, the organization inside the fire control computer begins to look like older systems with many "canned" delivery modes. Since limited computer storage is available for different modes, the approximations are patched up to work within certain limits (say a 100-foot error) over a fairly wide range. Thus the approximation to truly universal operation is achieved at some loss in accuracy even at those conditions for which the system is optimized.

In 1967, algorithms submitted in proposals by leading fire control vendors were evaluated at The Boeing Company. The best of these was embodied in a computer program and tested over a wide range of conditions, for both streamlined and drogued bombs. Sample results are shown in Tables I and II. No existing fire control system would include exactly these equation errors, but they are typical of the 1967 state-of-the-art. Note that these errors come from the equations alone, with no allowance for round-off errors, sensor errors or steering errors. It is clear that refinement could reduce the systematic errors, but not enough to make them insignificant.

Table I Typical Equation Errors – M117 Bomb

	FLIGHT PATH ANGLE AT RELEASE					
	45° DIVE		LEVEL		45° TOSS	
ALTITUDE (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)
2,000	1,885	−15	10,864	−330	28,559	−718
5,000	4,367	−37	16,853	−399	31,031	−905
10,000	7,832	−64	23,333	−528	34,626	−1,226
15,000	10,683	−71	28,035	−621	37,739	−1,524

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 101,
CONDITIONS: TARGET AT SEA LEVEL
NO WINDS
NO EJECTION VELOCITY
600 KNOTS RELEASE SPEED
+ MEANS CALCULATED RANGE IS TOO LONG
− MEANS CALCULATED RANGE IS TOO SHORT

Table II Typical Equation Errors – Mk 82S Bomb

	RELEASE SPEED					
	200 KNOTS		400 KNOTS		600 KNOTS	
ALTITUDE (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)	TRUE RANGE (FT)	EQUATION ERROR (FT)
500	1,507	−7	2,483	+78	3,111	+185
1,000	1,919	−11	3,010	+99	3,677	+329
2,000	2,360	−26	3,554	+101	4,255	+345
5,000	2,901	−91	4,244	+23	5,007	+267

REFERENCE: NWL BALLISTIC TABLE 010
CONDITIONS: LEVEL RELEASE
TARGET AT SEA LEVEL
NO EJECTION VELOCITY
NO WIND

2.0 NEW CALCULATION METHOD

2.1 Development

Research provided the clue which led to a calculation breakthrough. Work with mathematical series expansions showed better results if the equations were converted into a series with range, instead of time, as the independent variable. Applied to the numerical integration technique, this meant that instead of dividing the trajectory into segments (e.g., of 0.01-second duration) the trajectory would be divided into segments (e.g., covering 10 feet of downrange travel). The first experiments showed good accuracy (errors of a few feet or less) could be obtained with 10 or fewer integration steps, giving about a 100 to 1 improvement over the number of steps used to calculate the precise ballistic tables. Two years work produced refinements to extend the range of weapons and delivery conditions which could be handled by the same basic method. Later, other refinements increased the speed of computation. This development process is continuing, but the new algorithm is better in its current state than any computation method now used in an aircraft fire control computer.

2.2 Description

The new algorithm is accurate and flexible because most of the calculation is performed in the same manner that ballistic tables are calculated. At first, exactly the same equations were used to calculate atmospheric density, speed of sound, gravity acceleration, and drag coefficient. Later work has developed new equations which yield the same answers, but can be computed faster. At no point in the basic equations (the models of the physical world) has any approximation been used that is different from those accepted and used in the calculation of ballistic tables.

Figure 3 illustrates how the numerical integration process of the algorithm works and the essential differences between it and the process used in ballistic table calculation. Figure 3, Point A, shows a defined release point (speed, altitude, and dive angle). We know everything necessary about the release point. The impact range is to be calculated.

The range of the impact point must be estimated first so the size of the calculation intervals can be chosen. The range the bomb would go in a vacuum is the estimate used. This estimate only depends on the release conditions and is calculated easily with a single equation. (See Figure 3, Point B.)

Figure 3, Point C, shows the range estimate divided into five equal integration steps (an arbitrary number of steps based on experimental results). This division limits the amount of calculation required. If total calculation time is to remain small and fixed (a requirement for airborne use), the number of integration steps must remain small and fixed.

Integration is shown in Figure 3, Point D. Starting at the release point, the position, velocity, etc., of the projectile is calculated as it reaches the range corresponding to the end of the first step. (See "1," Point D.) No trajectory calculations are done at any point between the ends of the steps. To calculate "1," begin with the release point and calculate the average value of the forces acting on the projectile between release and "1." The simplest estimate would be that the forces are the same throughout the interval as at release. This estimate is good only if the interval is very short or if the forces change slowly. A better estimate is needed to use long, but few, steps. This is obtained through the Runge-Kutta technique of numerical integration. The one used in the algorithm is the "standard, fourth-order, Runge-Kutta technique."

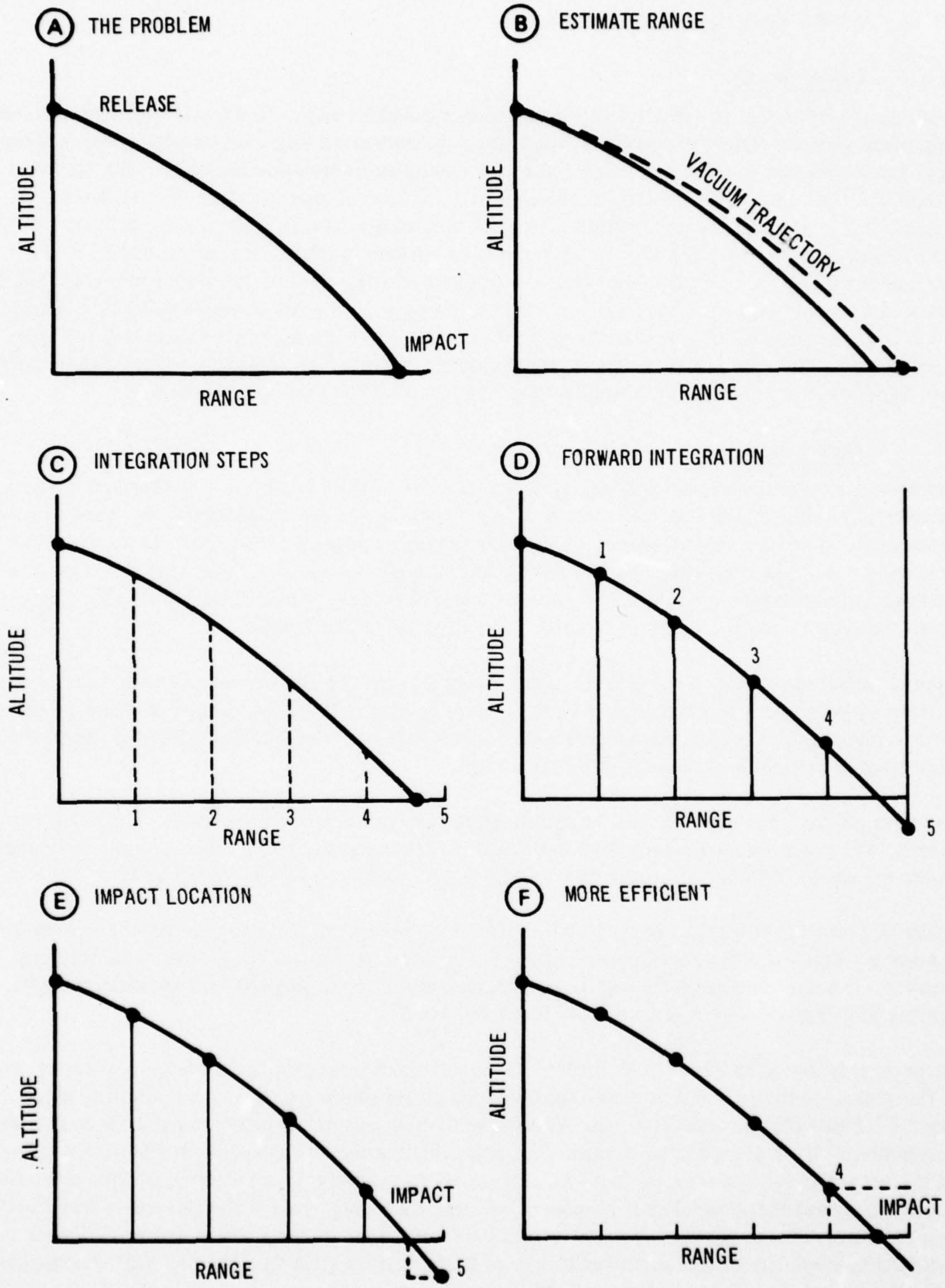

Figure 3. Numerical Integration

The average values of the forces in the interval, expressed as functions of range, are multiplied by the length of the step to find a close approximation of "1." The same practice is repeated to go from "1" to "2" and so on until "5" is reached.

At "5" the computer notices that the calculated position is under ground. It is not known where between "4" and "5" the projectile struck the ground. Recall that the original differential equations of motion were functions of time. These equations were rewritten to be functions of range. Now a new set is used, with the same equations rewritten to be functions of altitude. In altitude, it is easy to decide the required step length. The step length is just the difference between the altitude of "5" and the target altitude. (See Figure 3, Point E.) One more integration step, using the same integration technique with slightly different equations, carries the calculation to precisely the target altitude. This method of locating the impact point is an improvement over the way it is done in ballistic table calculations. It is obviously wasteful, though, to calculate forward to "5" and back to the impact or return to "4" and take a step down in altitude to the impact point. A set of logical tests was developed to detect the last point above ground and initiate the change to the altitude coordinate. (See Figure 3, Point F.) These tests also detect cases (as in drogued weapons) where the trajectory has become so steep that it is more accurate to take the remaining steps in altitude than in range.

The basic integration method applies to any ballistic projectile, but some adaptation is required for each type weapon. The adaptations included in the algorithm are described briefly below:

2.2.1 Streamlined Bombs

The process described fits streamlined bombs exactly. It is only necessary to provide the bomb size and a table describing how the drag coefficient of the particular bomb varies with Mach number.

2.2.2 Bullets

After being fired, a bullet is really just a small, streamlined bomb and is handled as such. The muzzle velocity of a particular gun-bullet combination is added to the aircraft velocity at release, as velocity imparted to a bomb by an ejector would be added. The weight and drag coefficient table must be provided as well as muzzle velocity, just as for a bomb.

2.2.3 Drogued Bombs

These bombs are more difficult to model accurately. Their common characteristic is that they are released in a relatively low drag configuration. At some time after release they deploy vanes or a parachute which greatly increases total drag. There is no basic difficulty in calculating bomb trajectory before the drogue is deployed. Nor is there a basic difficulty in calculating the trajectory after the drogue is deployed. The problem is to break the calculation correctly into two parts, one using each drag function.

This has been worked out very satisfactorily in the algorithm for the Mk 82S bomb. The time until the drogue opens is about half a second. The procedure used is: (1) Calculate the time of drogue deployment using the exact equations; (2) Use a simple approximation (Taylor's series expansion) to find bomb position and velocity at the time the drogue opens; and (3) Use the basic algorithm with the correct drag to integrate the drogued trajectory down to the point of impact.

Computation for other drogued bomb types, such as the Mk 43, will require adding appropriate logic for the particular drogue deployment mechanism.

2.2.4 Bluff Bombs

These are new bombs, such as the BLU-58, which may or may not be drogued. Bluff bombs are treated satisfactorily as streamlined or drogued bombs, depending on whether they are drogued.

2.2.5 Cluster Bombs

These weapons are released as fairly large, low-drag containers. At a predetermined point in their fall, they use some mechanism to dispense smaller weapons. These smaller weapons are higher drag than the container and may also develop lift. The fire control computer should predict the impact location of the center of the pattern rather than individual positions of the small bombs. The method of applying the algorithm is to calculate the trajectory of the container to the point where it dispenses the small weapons; switch to a drag function which describes the motion of the pattern center; and integrate down to the ground. Once again, the important factor is providing the logic to switch the drag function at the right place.

2.2.6 Rockets

Work is underway to extend the algorithm to include unguided rockets and tracer bullets. Rockets present several new problems. They change weight during flight; they have thrust (at varying levels) as well as drag; and they slew around just after firing because their launchers are not lined up with the aircraft direction of flight. It is felt that adequate provisions can be made for these problems in the algorithm.

2.3 Results

A characteristic of numerical integration techniques is that they can be made as accurate as desired by taking more integration steps. However, taking more steps requires a more expensive fire control computer. A standard suggested by R. Seeley of Naval Weapons Center, China Lake, has been adopted: The calculation error is insignificant if it is less than 10 feet or 1 milliradian of angle (0.06 degree) as measured from the release point. The arbitrarily chosen standard of five integration steps in range (plus one in altitude) provides answers which are almost always within this standard, even for release conditions far outside the usual precision delivery envelopes.

Before presenting the results obtained for some typical weapons, one characteristic of the tables must be explained. The differences between the calculated range and the reference range have been rounded to the nearest foot as done in the reference ballistic tables. These two rounding processes (in the ballistic tables and in the error calculation) introduce an uncertainty of about one foot into the errors given. In the case of small errors, this gives an erratic appearance to the numbers. The errors, measured in feet, are given as positive if the calculated range is longer than the reference and as negative if the calculation falls short. When the errors are converted to an angle for comparison with sighting errors, the absolute value is taken. The angles are rounded to the nearest tenth of a milliradian for the same reason that the linear errors are rounded to the nearest foot. Finally, the percentage error of the calculated time of fall is given. This can be important in operation, e.g., for wind correction.

Table III compares the new algorithm to that discussed earlier, using the M117 streamlined bomb. Table IV compares the two algorithms for the Mk 82S drogued bomb. Table V is for a bomb for which the calculated ballistic tables cover a wide range. The algorithm continues to give respectable results up to the table limits. Mach 2 at 70,000 feet is well outside the operational envelope of any system trying to precisely deliver conventional, unguided weapons. The generality of the method which achieves these results, however, provides the capability to handle the new weapons or new tactics not planned for when a system is built, but which become operational requirements.

Table VI shows the accuracy achieved with a typical gun and round combination. Actually, a single integration step gives fairly good answers.

These examples cover the range of general weapon types to which the algorithm has been applied. There should be no difficulty in obtaining similar results for other specific weapons of these general types.

Table III Algorithm Error Comparison – M117 Bomb

RELEASE ALTITUDE (FT) AND ALGORITHM	RELEASE ANGLE								
	45° DIVE			LEVEL			45° TOSS		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
2,000 OLD	−15	4.0	0.4	−330	5.6	2.3	−718	1.8	0.9
2,000 NEW	+1	0.3	0.0	0	0.0	0.0	−27	0.1	0.1
5,000 OLD	−37	4.2	0.2	−399	6.6	1.3	−905	4.7	1.3
5,000 NEW	0	0.0	0.0	−1	0.0	0.0	−38	0.2	0.1
10,000 OLD	−64	4.0	0.1	−528	8.4	0.8	−1,226	9.8	1.2
10,000 NEW	+3	0.2	0.1	0	0.0	0.0	−63	0.5	0.1
15,000 OLD	−71	3.1	0.3	−621	9.4	0.7	−1,524	14.4	1.2
15,000 NEW	+2	0.1	0.0	+2	0.0	0.0	−85	0.8	0.1

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 101, JULY 1967

CONDITIONS: 600 KNOT RELEASE SPEED
TARGET AT SEA LEVEL
NO WIND
NO EJECTION VELOCITY

Table IV Algorithm Error Comparison – Mk 82 Snakeye Bomb

RELEASE ALTITUDE (FT) AND ALGORITHM	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
500 OLD	–7	1.4	0.6	+78	5.9	1.5	+185	3.7	1.4
500 NEW	+1	0.2	0.0	0	0.0	0.0	0	0.0	0.0
1,000 OLD	–11	2.3	0.3	+99	9.5	0.9	+329	20.9	0.8
1,000 NEW	0	0.0	0.1	+1	0.1	0.0	0	0.0	0.0
2,000 OLD	–26	5.5	0.0	+101	11.9	0.7	+345	29.3	0.7
2,000 NEW	0	0.0	0.0	0	0.0	0.0	–1	0.0	0.1
5,000 OLD	–91	13.7	0.3	+23	2.7	1.0	+267	26.0	1.5
5,000 NEW	+1	0.2	0.0	–2	0.2	0.1	–1	0.1	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 010, OCTOBER 1964

CONDITIONS: LEVEL RELEASE
TARGET AT SEA LEVEL
NO WIND
NO EJECTION VELOCITY

Table V Algorithm Accuracy – Mk 76 Bomb

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	400			600			1,200		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	0	0.0	0.0	–1	0.0	0.0	–2	0.0	0.0
5,000	–2	0.1	0.1	–11	0.2	0.1	–40	0.5	0.1
10,000	–3	0.1	0.0	–22	0.5	0.0	–152	2.2	0.1
20,000	–19	0.5	0.0	–42	0.8	0.0	–91	1.2	0.1
30,000	–21	0.4	0.0	–48	0.8	0.0	–27	0.3	0.0
40,000	–22	0.4	0.1	–15	0.2	0.2	–39	0.4	0.3
50,000	–25	0.4	0.2	–182	2.2	3.1	–73	0.6	0.1
60,000	+24	0.3	0.1	–38	0.4	0.2	–36	0.3	1.1
70,000	+7	0.1	0.1	–79	0.7	4.6	–21	0.1	1.8

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 086, OCTOBER 1966

CONDITIONS: LEVEL RELEASE
NO WIND
TARGET AT SEA LEVEL
NO EJECTION VELOCITY

Table VI Algorithm Accuracy – M61 Gun with M56 Round

RELEASE ALTITUDE (FT)	RELEASE ANGLE								
	10° DIVE			15° DIVE			20° DIVE		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
500	0	0.0	0.0		NO REFERENCE VALUES				
750	–1	0.0	0.0						
1,000	–3	0.1	0.4*	0	0.0	0.7*	0	0.0	1.0*
1,250				–1	0.1	0.0	0	0.0	0.0
1,500		NO REFERENCE VALUES		–2	0.1	0.3*	0	0.0	0.5*
1,750							-1	0.1	0.4*

*TIME ERROR OF 0.01 SECOND

REFERENCE: ARMAMENT MEMORANDUM REPORT 64–5, FEBRUARY 1964

CONDITIONS: 400 KNOTS RELEASE SPEED
NO WIND
TARGET AT SEA LEVEL

3.0 OPERATIONAL SIGNIFICANCE

3.1 Attack Aircraft Application

The primary goal of the algorithm development is to apply it to aircraft intended to deliver unguided, nonnuclear weapons. The best estimates show that it will fit easily into the new fire control computers, such as that in the A-7E, in place of the calculation methods now used. More development is required to move the algorithm from the status of a research tool to an operational tool. When this is achieved, however, it will be possible to retrofit the algorithm into existing aircraft just by changing the computer program.

Preparation of this document is an early step in a contract with Naval Weapons Center, China Lake. The primary objective of the contract is to do the mathematical development work necessary to bring the algorithm to a point where it could be programmed for the A-7E fire control computer.

3.2 Contribution to System Performance

There is some confusion and controversy over the accuracy of a fire control system. A given aircraft, on a test range with the pilot's option of delivery conditions, might be able to deliver streamlined bombs with a 100-foot circular error probable (CEP). The same airplane may record an average target miss of 1,000 feet in North Vietnam. The amount by which the algorithm can improve the basic mechanical ability of the system can be estimated. What this improvement will amount to in a combat situation is difficult to establish.

An error in the fire control equations will result in a fixed-bias error. Over a reasonable range of conditions, however, this bias error will vary and may be treated—without gross injustice—as a random error. Figure 4 illustrates the contribution to total system error which would be made by a random equation error of 50 feet or 100 feet. There are two basic bars: one labeled 1968 technology; another labeled 1975 technology. These represent calculated weapon delivery errors which include all the estimated sensor errors and pilot errors (sighting, steering) for first quality systems of the indicated time periods. The conditions chosen for evaluation are shallow-dive deliveries of streamlined bombs with release about 8,000 feet from the target.

Extensions above these bars show how errors are increased if the fire control equations introduce random errors averaging 50 feet or 100 feet in size. The 100-foot figure is probably the better estimate of current calculation error. Random errors add according to the square root of the sum of the squares. Also, the errors introduced by computation are not the largest part of the total. The effect on system effectiveness is more apparent if one considers that the chance of killing a point target is generally proportional to one over the square of the CEP.

The change in system accuracy produced by switching to an algorithm which produces very small errors is significant. This improvement comes almost free compared to the cost of adding newly developed avionics. It is believed that the change in computation method will result in a system which also is less susceptible to combat degradation. The improved flexibility will permit the use of more varied tactics, which will contribute to improved aircraft survival. A better chance of survival should improve pilot performance—if only through practice. Also, the improved computation and display techniques using it can lighten the pilot's workload during the critical weapon delivery run.

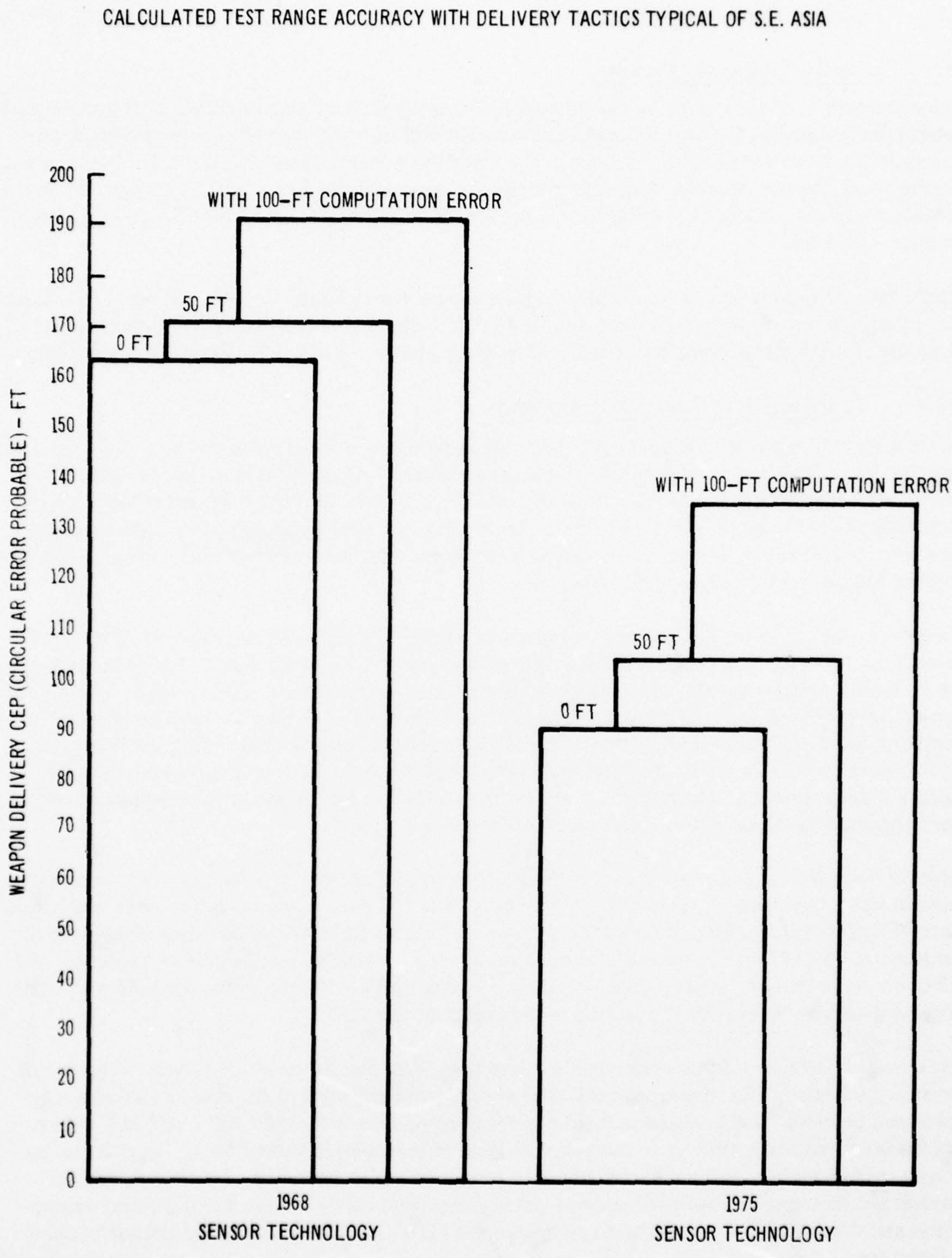

Figure 4. Contribution of Computation Errors to System Errors

PART II

TECHNICAL DESCRIPTION

1.0 INTRODUCTION

Part II presents the mathematical aspects of finding the impact range of a projectile. Major topics covered in this part are: (1) the basic mathematical model, (2) different coordinate systems, (3) integration process, (4) adaptations to various weapons, and (5) sample results. Coverage of these topics leads to an algorithm for calculating weapon impact ranges.

The basic technique used in the algorithm is to transform the differential equations so that down-range is the independent variable. Then the fourth-order, Runge-Kutta integration formulas are used to integrate these differential equations to obtain the impact range. Two variations of the basic technique are presented for special cases: (1) when the weapon trajectory becomes very steep, it is desirable to switch the integration process to a set of differential equations with altitude as the independent variable; (2) when dealing with time-fuzed dispenser weapons it is convenient to integrate the differential equations with time as the independent variable over the segment of trajectory between release and fuzing.

Application of the algorithm presented in this document depends on the availability of a current, advanced, airborne, digital computer. Also, it depends somewhat on the fact that the weapon impact range will be calculated repetitively. This process provides a good estimate of the integration interval size and, consequently, saves calculation time.

2.0 MATHEMATICAL MODELS AND THEIR INTEGRATION

The differential equations describing the motion of a projectile are discussed in this Section. These equations are given first with time as the independent variable. Then they are transformed twice; once to make the downrange, X, the independent variable and again to make the altitude, Y, the independent variable. Also discussed are the basic method used for integration and the comparative efficiencies resulting from the use of the different coordinate systems. In this section, the discussion is limited to streamlined bombs. Adaptations to other weapons are given in Section 3.0.

2.1 Basic Differential Equations

The development of an effective weapons release system is inherently dependent upon obtaining solutions of the equations for the motion of a projectile within the atmosphere. This is generally a difficult mathematical problem which has not been solved completely. The major difficulty stems from the nonlinearities introduced by the atmospheric effects on a falling weapon.

In choosing the mathematical model, two considerations have been kept in mind. The major objective of the mathematical analysis is to yield the weapon impact point. The main effect of this is that the weapon mass can be assumed to be a point mass. Also, the choice of the model is dictated by the need to evaluate results against some standard. Since the armed forces publish range tables for various weapons, the model is chosen to conform as closely as possible to the model used for these tables.

The equations of motion are developed assuming the projectile is a point mass acted on only by the force of gravity and the retardation forces due to air resistance. The trajectory can be restricted to a plane by ignoring crosstrack effects such as winds. For practical applications, the effect of winds can be accounted for in a straightforward manner.

The assumptions adopted are summarized below:

a. The Earth is flat and nonrotating.
b. The gravitational attraction is a function of altitude.
c. The projectile is a point mass.
d. The projectile is not powered and has a constant mass.

Under these assumptions, the differential equations of motion have the following form. (See Reference 1 for a complete discussion.)

$$\frac{d^2X}{dt^2} + H\frac{dX}{dt} = 0$$

$$\frac{d^2Y}{dt^2} + H\frac{dY}{dt} + g(Y) = 0 \qquad (2\text{-}1)$$

where X, Y, and t denote downrange, altitude, and time, respectively. The coefficient H, which is the drag function, is given by

$$H = \frac{\rho}{M} D^2 K_D V , \tag{2-2}$$

where ρ is the atmospheric mass density, M is the bomb mass, D is the bomb diameter, K_D is the weapon coefficient of drag, and V is the velocity. The atmospheric density ρ is given as a function of altitude which is fitted to measured values of atmospheric density. K_D, which is equal to $\frac{\pi}{8} C_D$, is empirically derived and given in tabular form as a function of mach number. The gravitational acceleration of a point mass above the surface of the Earth is given by

$$g(Y) = g_o \frac{R_E^2}{(R_E + Y)^2} \tag{2-3}$$

where g_o is the local value of gravity at sea level and R_E is the local value of the Earth's effective radius.

The above differential equations are not analytically integrable if an accurate model of H is used, because an accurate model would render them extremely nonlinear.

The two second-order, differential equations given in (2-1) will now be rewritten as four first-order, differential equations. This is done to get the differential equations in a form which is more suitable to the integration process used. Two new variables V_X and V_Y are defined by

$$\frac{dX}{dt} = V_X$$
$$\frac{dY}{dt} = V_Y \tag{2-4}$$

Substituting the above expressions in equations (2-1) results in

$$\frac{dV_X}{dt} = -H V_X$$
$$\frac{dV_Y}{dt} = -H V_Y - g(Y) \tag{2-5}$$

The four first-order, differential equations (2-4) and (2-5) are the desired equations with time as the independent variable.

2.2 Transformation of Basic Equations

For reasons which will become clear later in the document, it is desirable to transform the differential equations of motion into two other forms. One form uses downrange X as the independent variable (X-based), and the other form uses altitude Y as the independent variable (Y-based).

The terms X-based, Y-based, and time-based are adopted to ease the terminology in the text.

The transformation to make X the independent variable will be performed first. Multiplying the equations in (2-5) by $\frac{dt}{dX}$ or $\frac{1}{V_X}$

(Note: $\frac{dt}{dX} = \frac{1}{V_X}$),

$$\frac{d\,V_X}{dX} = -H \qquad (2\text{-}6)$$

$$\frac{dV_Y}{dX} = \frac{-H\,V_Y - g(Y)}{V_X} \qquad (2\text{-}7)$$

Similarly, the equations in (2-4) can be transformed to

$$\frac{dY}{dX} = \frac{V_Y}{V_X} \qquad (2\text{-}8)$$

and

$$\frac{dt}{dX} = \frac{1}{V_X} \qquad (2\text{-}9)$$

Equations (2-6) through (2-9) now represent a complete formulation of the ballistics equations in terms of the variables V_X, V_Y, Y and t, with X as the independent variable. Note that the equations in (2-6), (2-7), and (2-8) are uncoupled from (2-9), the equation involving time. Therefore, the first three equations (2-6), (2-7), and (2-8) can be solved simultaneously without solving for the variable t by equation (2-9). Figure 5 shows the salient features in the X-Y plane.

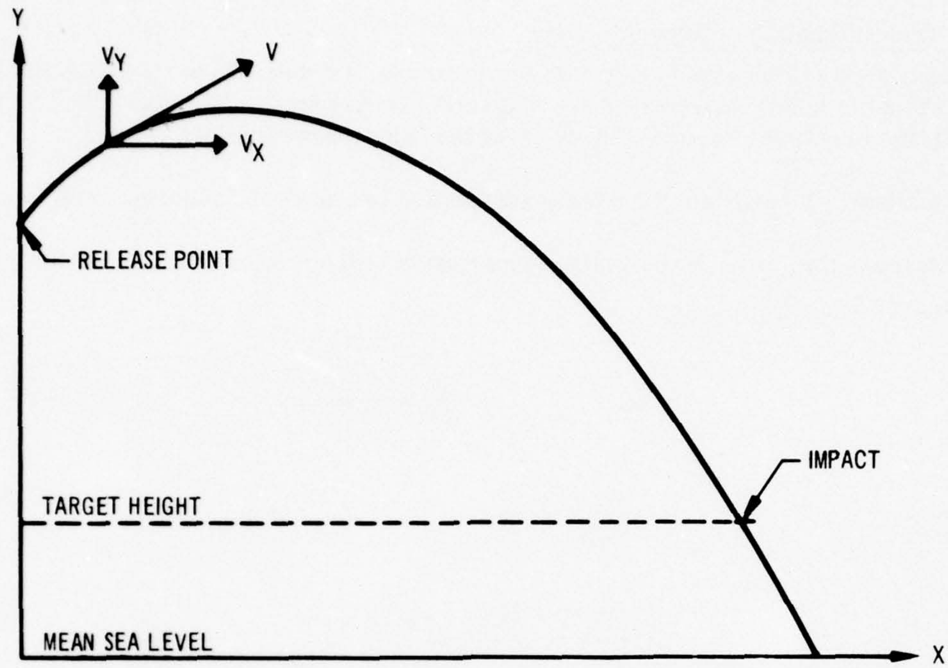

Figure 5. Salient Features in X-Y Plane

One can form a set of differential equations with Y as the independent variable in a similar fashion. Multiplying the equations in (2-5) by $\frac{dt}{dY}$ or $\frac{1}{V_Y}$ (Note: $\frac{dt}{dY} = \frac{1}{V_Y}$) results in

$$\frac{dV_X}{dY} = -H \frac{V_X}{V_Y} \tag{2-10}$$

$$\frac{dV_Y}{dY} = -H - \frac{g(Y)}{V_Y} \tag{2-11}$$

Also, from (2-4)

$$\frac{dX}{dY} = \frac{V_X}{V_Y} \tag{2-12}$$

and

$$\frac{dt}{dY} = \frac{1}{V_Y} \tag{2-13}$$

Equations (2-10) through (2-13) represent a complete formulation of the ballistics equations in terms of the variables V_X, V_Y, X and t, with Y as the independent variable. Note, as before, the equation involving time (2-13), is uncoupled from the other three equations.

Also note that the equations with X as the independent variable, (2-6) through (2-9), become indeterminate when V_X is zero (e.g., when the projectile is coming straight down), because V_X appears in the denominator. Similarly, the equations with Y as the independent variable, (2-10) through (2-13), become indeterminate when V_Y is zero (e.g., at level release). Consequently, the two sets of equations must be used accordingly.

The basic set of differential equations in the integration procedure developed here is X-based. This choice was based on experimental evidence which showed that this set of equations gave the most favorable results. A sample of this experimental evidence is given in Section 2.4. The other two sets of equations are used in special cases. When time fuzing is involved, it is convenient to use the time-based equations for the part of the trajectory between release and fusing. Y-based equations work best when the trajectory is very steep; i.e., the projectile is coming almost straight down.

2.3 Integration Procedure

As pointed out earlier, the differential equations describing a projectile's trajectory are not analytically integrable if an accurate model of function H is used. Consequently, a numerical integration procedure must be used. The approach taken in the generation of ballistic tables is to integrate the differential equations numerically, using time as the independent variable. Many steps are taken to insure the desired accuracy. (A typical step length is 0.01 second.) This method is accurate and flexible, but requires too much computer time to be suitable for weapon delivery application.

The procedure which proved to be most successful in eliminating this shortcoming is to use the fourth-order, Runge-Kutta integration formulas with the X-based, differential equations to integrate the trajectory to the vicinity of the impact point. Integration of the Y-based equations is then used to locate the impact point precisely, since its Y-coordinate is known. An estimate of the required computer time and memory for using this procedure to calculate a weapon impact point is given in Section 4.1.

Time-based and Y-based equations are used with the X-based differential equations in special cases. For example, it is convenient to use the time-based equations for the powered segment of unguided rocket trajectory and for the first segment of the time-fuzed dispenser weapon trajectory. It is desirable to switch to the Y-based equations when a weapon trajectory becomes very steep like at the end of the long trajectory of a drogued bomb.

Details of the integration procedure and the fourth-order, Runge-Kutta formulas are described in Section 2.3.1. Application of these formulas to the X-based, differential equations is described in Section 2.3.2.

2.3.1 The Runge-Kutta Integration Formulas

The Runge-Kutta formulas provide a step-by-step method of finding dependent variable values at given intervals of the independent variable. That is, values of the dependent variables at a particular value of the independent variable are found in terms of their values at the previous step.

Knowing the differential equations of a given system and the initial conditions, this process may be continued until the desired range is covered. This integration method requires no preliminary differentiation of the dependent variables beyond those prescribed. This makes it particularly useful if certain coefficients in the differential equations are empirical functions for which analytical expressions are not known.

The standard, fourth-order, Runge-Kutta formulas, which yield very good results even for coarse integration intervals, are described for a system of three ordinary differential equations. Consider the system of differential equations.

$$\frac{dY}{dX} = F_1 \quad (X, \; Y, \; V_X, V_Y)$$

$$\frac{dV_X}{dX} = F_2 \quad (X, \; Y, \; V_X, V_Y) \qquad (2\text{-}14)$$

$$\frac{dV_Y}{dX} = F_3 \quad (X, \; Y, \; V_X, V_Y).$$

For example, if the X-based equations describing a projectile's trajectory were to be solved, then the above equations would be of the following form.

$$\frac{dY}{dX} = F_1 = \frac{V_Y}{V_X}$$

$$\frac{dV_X}{dX} = F_2 = -H \qquad (2\text{-}15)$$

$$\frac{dV_Y}{dX} = F_3 = \frac{-HV_Y - g(Y)}{V_X}$$

(See equations (2-6), (2-7), and (2-8))

If h is the integration interval, the $(n+1)$th values of Y, V_X, V_Y are computed from the nth values by

$$Y_{n+1} = Y_n + \frac{1}{6}(K_o + 2K_1 + 2K_2 + K_3)$$

$$V_{X_{n+1}} = V_{X_n} + \frac{1}{6}(M_o + 2M_1 + 2M_2 + M_3) \qquad (2\text{-}16)$$

$$V_{Y_{n+1}} = V_{Y_n} + \frac{1}{6}(N_o + 2N_1 + 2N_2 + N_3)$$

where

$$K_o = h F_1 \quad (X_n, Y_n, V_{X_n}, V_{Y_n})$$

$$K_1 = h F_1 \quad (X_n + \frac{1}{2} h, Y_n + \frac{1}{2} K_o, V_{X_n} + \frac{1}{2} M_o, V_{Y_n} + \frac{1}{2} N_o)$$

$$K_2 = h F_1 \quad (X_n + \frac{1}{2} h, Y_n + \frac{1}{2} K_1, V_{X_n} + \frac{1}{2} M_1, V_{Y_n} + \frac{1}{2} N_1)$$

$$K_3 = h F_1 \quad (X_n + h, Y_n + K_2, V_{X_n} + M_2, V_{Y_n} + N_2) \tag{2-17}$$

and the values of M_o , M_1 , M_2 , and M_3 are computed by exactly the same formulas as the correspondingly subscripted K's , except that F_2 is used instead of F_1 . Similarly, N_0 , N_1 , N_2 and N_3 are computed using F_3 instead of F_1 .

The fourth-order, Runge-Kutta formulas can be expanded in a similar fashion to a system of four first-order, differential equations.

When applying numerical integration formulas, it is necessary to decide the size of the integration steps. For this problem, where very few integration steps are taken (i.e., on the order of five), the step size depends largely on the impact range of the specific trajectory. Therefore, it is necessary to have an estimate of the impact range before starting integration.

One way of obtaining this estimate is to use the vacuum trajectory in finding the impact range. This works well for low-drag bombs; however, this estimate may be off by as much as a factor of two or three in the high-drag bomb case. Estimating the impact range will not be a problem in practice. Since the impact range will be calculated repetitively, one will always have a good estimate of the impact range once the process is started.

2.3.2 Integration Algorithm

Originally, the integration was performed in the following manner. The fourth-order, Runge-Kutta formulas were applied to the X-based, differential equations, (2-6), (2-7), and (2-8), with a test performed after each integration step to determine whether the bombs were below target altitude. When the bomb was found to be below this altitude, a switch was made to the Y-based equations, (2-10), (2-11), and (2-12). Then a single integration step was taken to the target since the integration step size in the Y-based system was known exactly. (See Figure 6.) Using this procedure, it was found that the solution accuracy for drogued weapons degraded rapidly as release altitude went above 2,000 feet. V_X was so close to zero at impact that some of the approximations internal to the Runge-Kutta process had V_X either zero or negative. As V_X appears in the denominator, it renders the expression indeterminate. The problem was resolved by adding two logical tests to the calculations.

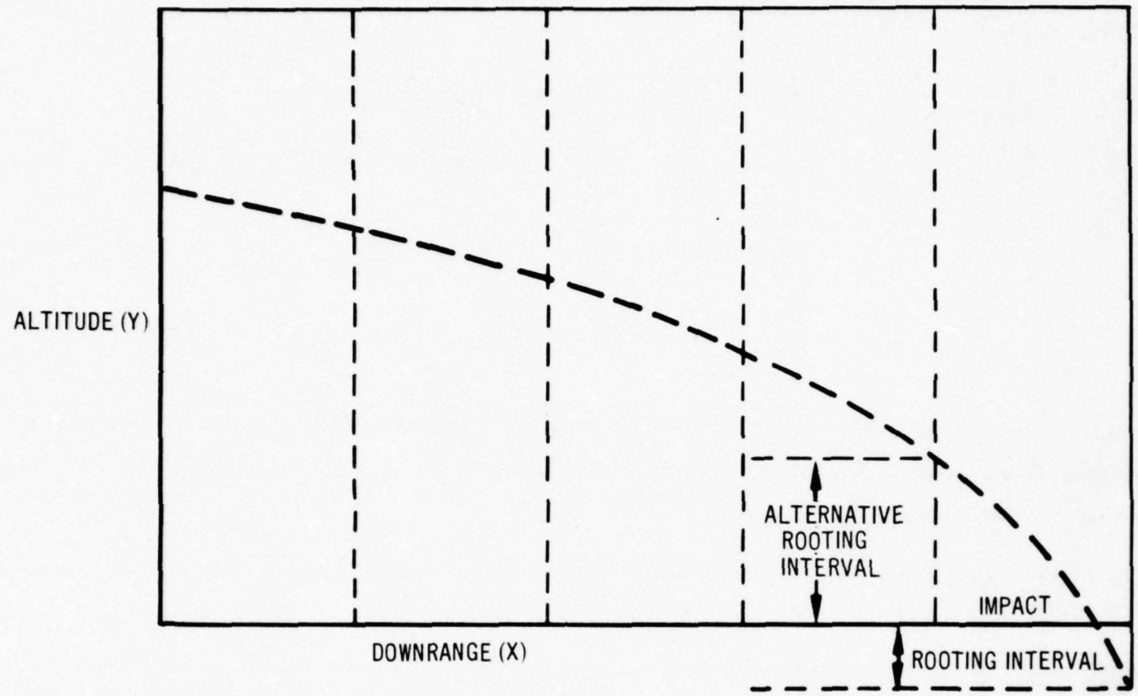

Figure 6. Equation Rooting - Integration in Y

Coordinates are switched if the vertical distance the bomb fell in the last downrange integration step is greater than the distance to the ground. If the next integration step would put the bomb far below the ground, this replaces a long step in X and a long step in Y with a short step in Y. Calculation time is saved and accuracy is improved.

The other test is on the ratio of velocity components. When the vertical velocity is larger than the horizontal velocity, the coordinates can be switched to advantage. Different steepness criteria, ranging from 1 to 5, have been tried. Now the test is of this form:

$$\text{If } \frac{-V_Y}{V_X} \geqslant 2, \text{ switch coordinates.}$$

Since this switch may take place some distance from the impact point, it is sometimes necessary to take more than one step in Y.

A very simple method is used now. If the estimated range was divided into N_X steps and "i" is the number of steps taken so far, then the number of Y steps to take is:

$$M_Y = N_X - i + 1, \text{ if } (N_X - i) > 0, \text{ or}$$

$$M_Y = 1, \qquad \text{if } (N_X - i) \leqslant 0.$$

The addition of these two tests has made it possible to calculate Snakeye trajectories from releases as high as 70,000 feet without encountering singularities.

One final topic concerning the integration algorithm needs to be covered. This concerns the question of how many integration steps are required in the process of solving for the weapon impact point. The final result is more accurate with a large number of integration steps, but computer time is increased. The final results may not be sufficiently accurate if too few steps are taken. Therefore, when choosing the number of integration steps to be used, a trade-off exists between accuracy and computation time.

It was decided on the basis of experimental results to use five integration steps now. A typical example of these results is given in Figure 7. This number of steps is somewhat arbitary and could be adjusted easily for specific applications.

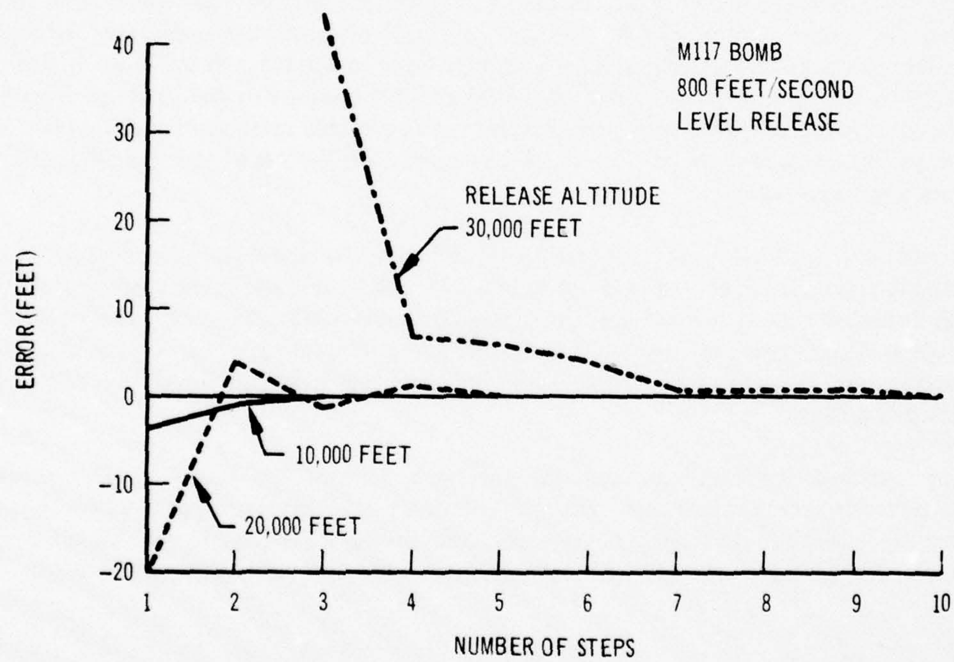

Figure 7. Integration Convergence

2.4 Comparative Efficiencies of Coordinate Systems

The discovery leading to the development presented in this document was that the Taylor's series with downrange as the independent variable (downrange-base) converged faster than that with time as the independent variable (time-base). However, no work was done to demonstrate directly that traject ry integration converged more rapidly with downrange as the independent variable. Since there are advantages to using time as the independent variable, the comparative efficiency of the two approaches was investigated.

A computer program was devised to calculate weapon impact points with the X-based and time-based equations using identical, fourth-order, Runge-Kutta, integration formulas. Pains were taken to ensure exact comparability of results to cases using the standard algorithm. The Mk 82S drogued bomb was the primary weapon tested. A comparison of results is given in Figure 8. The magnitude of the error in the computed range is plotted for a given number of integration steps. The two solid lines compare the errors for release at a 500-foot altitude; the dotted lines are for release at a 3,000-foot altitude. Note that the errors are plotted on a logarithmic scale. Time-based integration for this weapon requires almost three times as many integration steps to achieve the same accuracy. This comparison holds over a wider range of release conditions.

Two effects contribute to the difference in the case of high drag bombs: (1) the bomb's deceleration is more nearly constant when considered as a function of downrange, making it easier to approximate its average over an integration step; (2) due to comparative placement of the integration steps, the bomb is going much faster at release than at impact and travels much more than one n-th of its trajectory in the first one n-th of its time of fall. The first part of the trajectory also displays the highest deceleration rates. Therefore, the time-based integration takes longer steps in the more critical portion of the trajectory than does downrange-based integration with the same number of steps. This factor in the accuracy difference could be moderated greatly with a suitable method of picking unequal length time steps. However, a number of complexities arises with the use of unequal steps and this approach has not been pursued.

Also some comparative evaluation was done for the M117 streamlined bomb. This weapon slows down very little during fall, so the forces are more nearly constant and the integration intervals of the two methods almost match. As a result, the comparative advantage of downrange-based integration is not as great, although it remains superior to time-based integration. This leads to a possibility that time-based integration may prove better overall for some special applications, such as streamlined, time-fused cannisters.

Over the entire spectrum of trajectory calculation problems, it seems that downrange-base integration has about a two to one average advantage. Different methods have different logical advantages when dealing with special situations such as altitude-fusing, time-fusing, loft-delivery, etc. To achieve the optimum for specific weapons one may have to use a combination of coordinate systems.

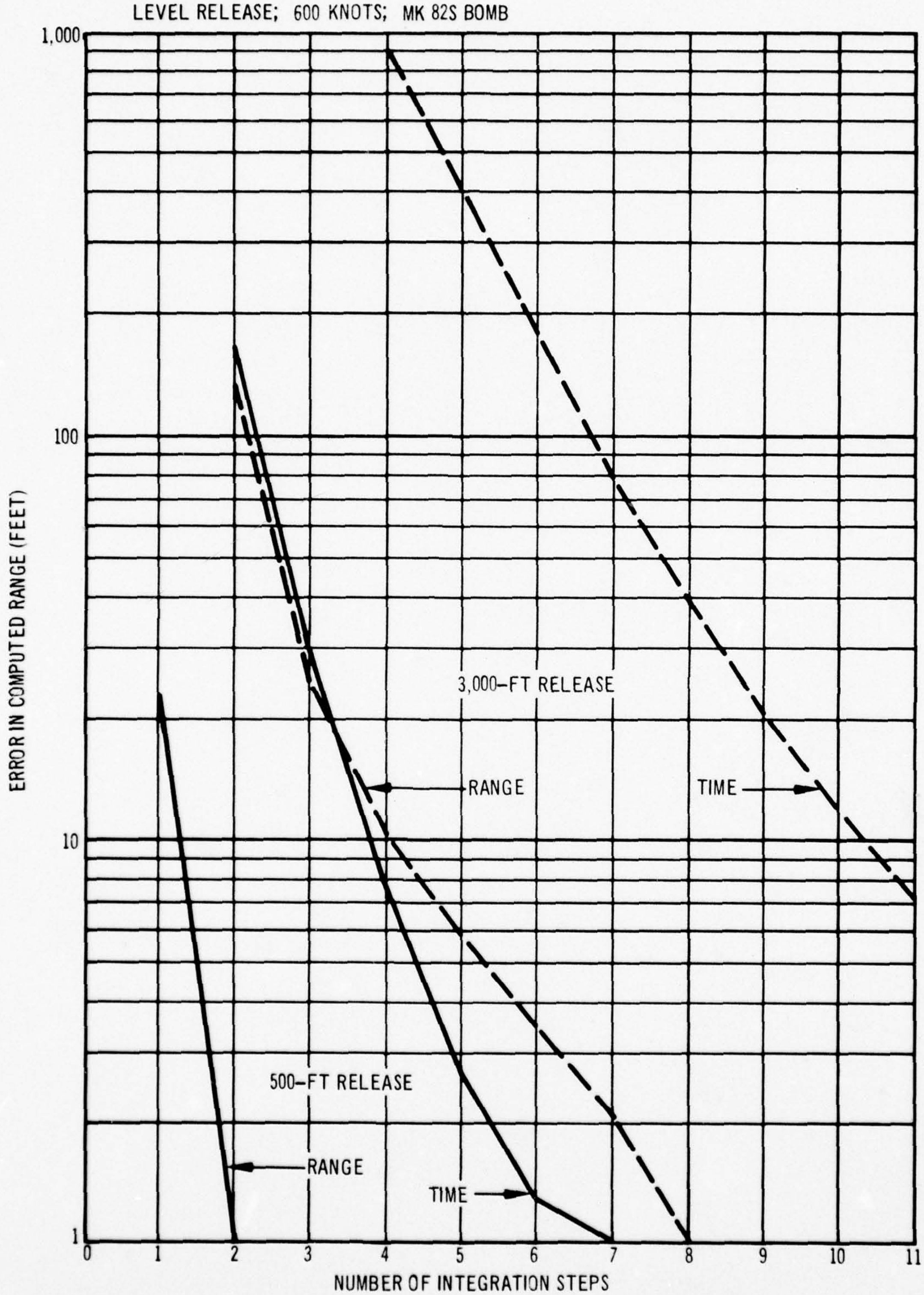

Figure 8. Convergence Comparison Range - Based and Time-Based Integration

3.0 ADAPTATIONS TO VARIOUS WEAPONS AND SAMPLE RESULTS

This section contains the mathematical/logical adaptations necessary to calculate impact points for various weapons. Also, sample results and qualitative descriptions of these are given for a variety of weapon types. The weapon types covered are: (1) streamlined bombs, (2) guns, (3) drogued bombs, (4) cluster bombs, and (5) unguided rockets.

The tabular form in which the sample results are presented requires some explanation. There are three entries for each identified set of release conditions. The first entry is the "range error" in feet. This is the difference between the weapon range given in the ballistic table and the range computed by the algorithm. A negative sign on the error means that the computed range was smaller than the table range. The next entry is "angular error" in milliradians. This converts the range error into the equivalent angular error at the release point. This angular form is convenient for comparison with other fire control system errors, most of which are basically angular. A good rule of thumb is that if the range error is less than 10 feet or the angular error is less than 1 milliradian the computation error will not significantly degrade the overall system accuracy. The third entry is the "time error" in percent. By including the uncoupled time equation in the integration, the time of fall can be computed quite accurately. Ability to compute the time of fall accurately will become important when the algorithm is put to an operational application. The target for all tables is at sea level and there is no wind; this matches the conditions in the ballistic tables.

3.1 Streamlined Bombs

Calculation of streamlined bomb impact ranges does not require any adaptation of the process described in Section 2.3.2. Sample results comparing the impact ranges calculated by the algorithm to those from Navy ballistic tables are given in Tables VII through IX for the M117, Mk 82, and Mk 76 bombs, respectively. Table X gives results outside the normal A-7E operating region for the Mk 76 bomb. The tables show that the algorithm yields good results over a large range of release conditions.

3.2 Guns

The approach taken with guns is to treat them as streamlined bombs, handling the muzzle velocity as a large ejection velocity. Sample results for the M61 gun are given in Table XI. The calculation errors are defined as they were defined for bombs in the previous section. These sample calculations show that the bombing algorithm produces excellent results without any added logic or calculation.

3.3 Drogued Bombs

One of the major successes of the algorithm was successful calculation of the impact point of a drogued bomb. These bombs are dropped in a streamlined configuration and then vanes or a parachute is deployed to greatly increase their drag and retard their forward motion. Previous methods were marginal in working with high-drag projectiles and were unable to cope with the change in the drag function. Presently, drogued bombs are handled by splitting the trajectory into two parts, using a simple approximation for the streamlined portion and the algorithm for the drogued portion.

Table VII M117 Bomb Computation Accuracy

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	+1	0.1	0.0	0	0.0	0.1	0	0.0	0.0
3,000	0	0.0	0.0	+1	0.0	0.0	0	0.0	0.0
5,000	+1	0.1	0.0	+1	0.0	0.0	−1	0.0	0.0
7,000	+1	0.1	0.0	+2	0.1	0.0	−2	0.0	0.0
9,000	+1	0.1	0.0	+1	0.0	0.0	+1	0.0	0.0
11,000	+1	0.1	0.0	+2	0.1	0.0	−2	0.0	0.0
13,000	+1	0.1	0.0	+1	0.0	0.0	−4	0.1	0.0
15,000	+1	0.0	0.0	+2	0.0	0.0	+2	0.0	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 101, JULY 1967

CONDITIONS: LEVEL RELEASE
NO WIND
TARGET AT SEA LEVEL
NO EJECTION VELOCITY

Table VIII Mk 82 Bomb Computation Accuracy

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	0	0.0	0.0	+1	0.0	0.0	+1	0.0	0.0
3,000	0	0.0	0.0	+1	0.0	0.0	0	0.0	0.0
5,000	+1	0.1	0.0	+1	0.0	0.0	0	0.0	0.1
7,000	0	0.0	0.0	+1	0.0	0.0	+2	0.0	0.0
9,000	0	0.0	0.0	0	0.0	0.0	+1	0.0	0.0
11,000	+1	0.1	0.0	0	0.0	0.0	+1	0.0	0.0
13,000	+1	0.1	0.0	+1	0.0	0.0	0	0.0	0.0
15,000	+1	0.0	0.0	0	0.0	0.0	−1	0.0	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 169, MARCH 1969

CONDITIONS: LEVEL RELEASE
TARGET AT SEA LEVEL
NO WIND
NO EJECTION VELOCITY

Table IX Mk 76 Bomb Computation Accuracy

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	0	0.0	0.0	0	0.0	0.0	1	0.0	0.0
3,000	+1	0.0	0.0	−1	0.0	0.0	−6	0.1	0.1
5,000	+1	0.0	0.0	−2	0.1	0.1	−11	0.2	0.1
7,000	0	0.0	0.0	-2	0.1	0.0	−15	0.3	0.0
9,000	+1	0.1	0.0	−3	0.1	0.0	+1	0.0	0.0
11,000	0	0.0	0.0	−4	0.1	0.0	−26	0.5	0.0
13,000	+1	0.0	0.0	−6	0.2	0.0	−34	0.7	0.1
15,000	0	0.0	0.0	−9	0.3	0.1	−39	0.8	0.1

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 086. OCTOBER 1966
CONDITIONS: LEVEL RELEASE
NO WIND
TARGET AT SEA LEVEL
NO EJECTION VELOCITY

Table X Mk 76 Bomb Computation Accuracy In Extended Delivery Regime

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	600			900			1,200		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
10,000	−22	0.5	0.0	+4	0.1	0.1	−152	2.2	0.2
20,000	−42	0.8	0.0	−43	0.7	0.1	−91	1.2	0.3
30,000	−48	0.8	0.0	−82	1.1	0.0	−27	0.3	0.0
40,000	−15	0.2	0.2	−11	0.1	0.3	−39	0.4	0.3
50,000	−182	2.2	3.1	−123	1.3	1.9	−73	0.6	0.1
60,000	−38	0.4	0.2	+28	0.2	0.2	−36	0.3	1.1
70,000	−79	0.7	4.8	−17	0.1	5.3	−21	0.1	1.8

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 086, OCTOBER 1966
CONDITIONS: LEVEL RELEASE
NO WIND
TARGET AT SEA LEVEL
NO EJECTION VELOCITY

Table XI M61 Gun With M56 Round Computation Accuracy

DIVE ANGLE (DEG)	RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS) 300			450			600		
		RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
15	1,000	−1	0.1	0.6*	−1	0.1	0.0	−1	0.1	0.0
15	1,250	−1	0.1	0.5*	−2	0.1	0.5*	−1	0.1	0.5*
15	1,500	−2	0.1	0.3*	−2	0.1	0.8	−2	0.1	0.4*
20	1,000	0	0.0	3.0	0	0.0	0.0	0	0.0	1.1*
20	1,250	0	0.0	0.0	0	0.0	0.7*	0	0.0	0.0
20	1,500	−1	0.1	0.5*	0	0.0	0.6*	−1	0.1	0.7*
20	1,750	−1	0.1	0.4*	−1	0.1	0.0	−1	0.1	0.5

*TIME ERROR OF 0.1 SECOND

REFERENCE: ARMAMENT MEMORANDUM REPORT 64–5, FEBRUARY 1964

CONDITIONS: TARGET AT SEA LEVEL
NO WIND
3,300 FT/SEC MUZZLE VELOCITY

As noted before, the treatment of retarded weapons is complicated by the discontinuity in the drag coefficient at drogue deployment. If it is assumed that the deployment occurs instantaneously at a time t_D which is known or can be calculated in advance, the differential equations (2-1) have discontinuous coefficients at t_D due to the change in the ballistic drag coefficient K_D. Corresponding to t_D is point X_D at which (2-6) and (2-7) have discontinuous coefficients. In this case, it is best to split the integration domain into two parts corresponding to the different drag regions. (See Figure 9.)

If the drogue deployment occurs shortly after release as for the Mk 82 Snakeye drogued bomb, this difficulty can be overcome easily. Since X_D is very small compared to X_R and the bomb has very little drag in the first region, it is sufficient to integrate from release to X_D in one step. Normally, t_D is given as a function of the dynamic air pressure. The point X_D and the values of V_{XD}, V_{YD}, and Y_D must be computed before the Runge-Kutta procedure can be used to integrate from X_D to X_R. It is probable that due to shortness of t_D (0.5 second) the vacuum solution would suffice. However, it is not difficult to account for most of the drag effects in the interval $(0, X_D)$ as follows.

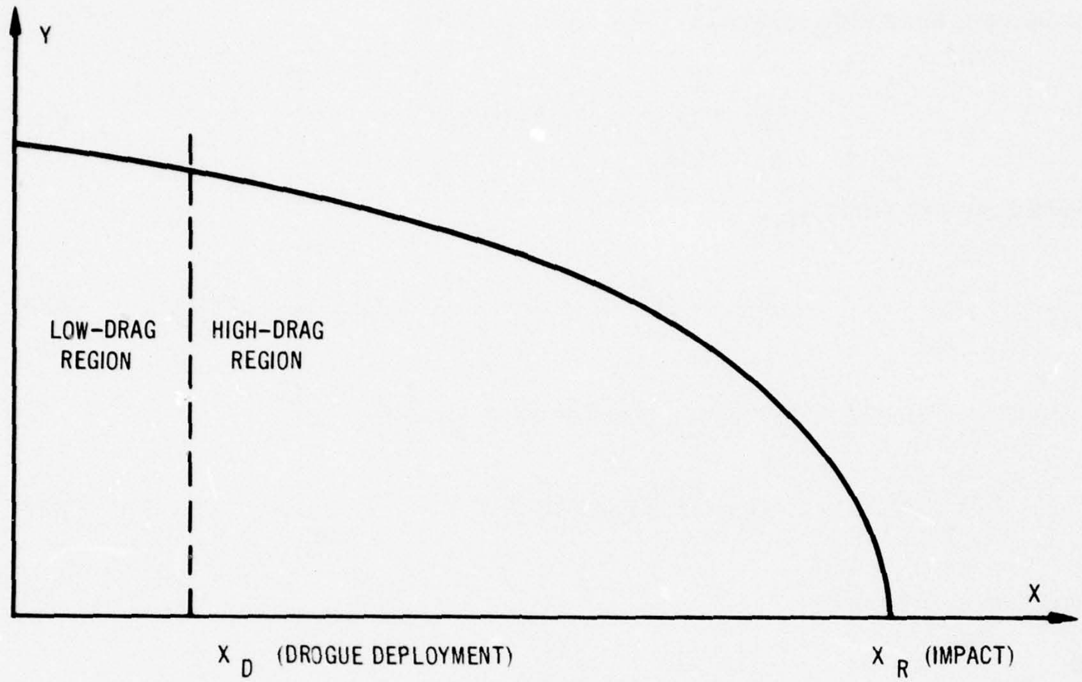

Figure 9. Drag Discontinuity

X_D is expanded first, in a power series in t about the point t = 0. That is:

$$X(t) = X(0) + \dot{X}(0)t + \frac{\ddot{X}(0)}{2} t^2 + \ldots \qquad (3\text{-}1)$$

The coefficients are computed readily from the release conditions. Denoting the release conditions by the subscript zero,

$$X(0) = 0, \quad \dot{X}(0) = V_{X0}, \quad \ddot{X}(0) = -H_0\dot{X}(0) = -H_0V_{X0} \qquad (3\text{-}2)$$

Retaining three terms in the series and evaluating at t_D yields

$$X_D = V_{X0} t_D (1 - .5 \; H_0 t_D). \tag{3-3}$$

A similar expansion yields Y_D,

$$Y_D = Y_0 + t_D \; (V_{X0} \; - .5 t_D (H_0 V_{Y0} + g(Y_0)). \tag{3-4}$$

Differentiating (3-1) with respect to t and evaluating at t_D gives

$$V_{XD} = V_{X0} (1 - H_0 t_D). \tag{3-5}$$

Similarly, V_{YD} is given by

$$V_{YD} = V_{Y0} - t_D \; (H_0 V_{Y0} + g(Y_0)). \tag{3-6}$$

The initial value of X_D, Y_D, V_{XD}, and V_{YD} needed for the integration process over the high-drag region is given by (3-3) through (3-6), respectively. The integration over this region is performed as described in Section 2.0.

Sample results for the drogued Mk 82S bomb are given in Table XII.

3.4 Cluster Bombs

A cluster bomb, which is a dispenser containing many individual bombs, is generally released in a streamlined shape. Then, at some predetermined condition (altitude, time after release, slant range, etc.), the bombs are released from the dispenser. The bombs have a much higher drag-to-mass ratio than the dispenser and slow down rapidly. Ballistics agencies have developed drag tables for a fictitious bomb which simulates the characteristics of the center of the pattern as though it were a single bomb.

Table XII Mk 82S Bomb Computation Accuracy

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	+1	0.3	0.0	+1	0.2	0.0	0	0.0	0.0
2,000	+1	0.3	0.0	0	0.0	0.0	0	0.0	0.0
3,000	0	0.0	0.0	0	0.0	0.1	−1	0.1	0.0
4,000	+1	0.2	0.0	0	0.0	0.3	−1	0.1	0.1
5,000	0	0.0	0.0	−1	0.1	0.0	−3	0.3	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 010, OCTOBER 1964

CONDITIONS: 20°DIVE
TARGET AT SEA LEVEL
NO WIND
NO EJECTION VELOCITY

Like drogued bombs, the cluster bombs are modeled by breaking the trajectory into two parts. The first describes the trajectory of the streamlined dispenser and the second describes the trajectory of the center of the pattern of bombs released by the dispenser. The simple approximation used for the first trajectory segment of the drogued bombs is not adequate for cluster bombs because the trajectory segment is much longer; therefore, a regular integration process must be used. The sample results given in Table XIII for the CBU-24B/B weapon are quite good, considering that the pattern is about 1,000 feet in diameter.

Two types of fuzing systems are considered here: one is altitude sensitive, the other is time sensitive. The basic coordinate system of the algorithm (i.e., the X-based system) works very well for calculating both segments of the trajectory of the altitude-fuzed weapon. During the first segment of the trajectory for this case, a hypothetical target is assumed at the fuzing altitude and the calculations are performed conventionally. For Table XIII, two range steps and one altitude integration step were used. Next, the same integration process was repeated for the second segment of the trajectory which is between the fuzing point and the actual target. For Table XIII, three range steps and one altitude step were used. In other words, two conventional trajectory calculations are performed to find a single impact point of an altitude-fuzed cluster bomb.

The same method cannot be used for time-fuzed cluster bombs. Normally, the time of fall is not calculated when the X-based equations are used. The approach taken is to use the time-based differential equations for the first segment of the trajectory. This is a simpler alternative than an elaborate rooting scheme. The first portion of the trajectory can be integrated accurately because it is short and low drag. The X-based equations are used for the second segment of the trajectory.

Table XIII CBU-24 B/B Cluster Bomb Computation Accuracy

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	420*			500			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
5,000	-2	0.2	0.3	-7	0.6	0.5	-33	2.8	1.5
6,000	-2	0.2	0.3	-8	0.6	0.4	-28	2.0	1.3
7,000	-2	0.1	0.2	-9	0.6	0.5	-25	1.5	1.1
8,000	-3	0.2	0.3	-9	0.5	0.5	-22	1.2	1.0
9,000	-4	0.2	0.2	-11	0.6	0.6	-21	1.0	1.0
10,000	-5	0.3	0.3	-11	0.5	0.6	-19	0.9	0.9

*400-KNOT SPEED NOT IN TABLE FOR HIGHER ALTITUDES

REFERENCE: F-105D BALLISTIC TABLES CBU-24 B/B OR CBU 29 B/B, FEBRUARY 1968

CONDITIONS: 30° DIVE
TARGET AT SEA LEVEL
NO WIND
6 FT/SEC EJECTION VELOCITY
2,500 FT FUSE ALTITUDE

3.5 Unguided Rockets

The unguided rocket is more difficult because of the added problems of thrust and change of mass. These functions are provided as tables with time as the independent variable. Linear interpolation is used for values between points in the tables. Since the primary variables are given as a function of time, it is more convenient and accurate to use the time-based differential equations for the powered segment of the trajectory. The X-based equations are used for the rest of the trajectory.

The necessary adaptations for the powered phase are covered next. At any time t_R, the acceleration due to propulsion is

$$A_T = \frac{F_T}{M_R}$$

Where

F_T = Rocket thrust force at time t_R

M_R = Rocket mass at time t_R

To form a function comparable to the aerodynamic drag function, H , we compute

$$H_R = \frac{V_R}{V}$$

The total drag function is

$$H = H_A + H_R,$$

where H_A is the drag function for unpowered weapons (i.e., for unpowered weapons $H = H_A$).

A program has been written to calculate impact ranges for unguided rockets. However, ballistic tables which can be used to compare results are not available at this time.

3.6 Sensitivity to Integration Interval Size

Results given in the preceding sections are based on dividing the exact trajectory length into the appropriate number of integration steps. The argument was that this is valid because the trajectory length changes little from one calculation to the next; therefore, use of the previously calculated range would yield almost the same integration steps. This section deals with the validity of this assumption. In Section 3.6.1, the maximum deviation of the release conditions which might be achieved by maneuvers during a single calculation interval is estimated. The corresponding effect of these deviations of the impact range is found in Section 3.6.2. This is done for a low-drag bomb (M117) and a drogued bomb (Mk 82S) for short, medium, and long trajectories. The maximum percentage impact range deviation due to maneuvers is estimated on the basis of this data. Finally, these deviations are included in determining the integration interval to demonstrate that they do not affect the results adversely.

3.6.1 Maximum Deviation in Release Conditions Due to Maneuvers

The following assumptions were made in estimating maximum deviation possible in release conditions during a single calculation interval.

a. The A-7 aircraft is the aircraft of interest.
b. The calculation interval is 0.1 second.
c. Only positive-load factors are considered, since weapons are not released under negative-load-factor conditions.

The release conditions affecting the impact range are altitude, velocity, and dive angle. Considering velocity first, it can be reasoned that the change in the magnitude of velocity in 0.1 second is negligible. Consider a hypothetical example where an accelerating force equal to the aircraft weight is applied. That is, this force exceeds that needed to overcome drag and lift. This accelerating force would change the velocity magnitude in 0.1 second by

$$\Delta V = a(0.1),$$

where a is the acceleration, and is equal to the gravitational acceleration for the chosen example. Therefore,

$$\Delta V = 3.2 \text{ ft/sec}$$

If the average bomb release velocity is assumed to be 400 knots, then the above ΔV would be less than 0.5% of this velocity. Since the available accelerating force is normally a small fraction of that assumed in the hypothetical example, it can be reasoned that the corresponding velocity deviation in 0.1 second is negligible.

The maximum deviation in dive angle due to maneuvers is estimated next. The deviation in direction an object is traveling during a time interval Δt at a velocity V and pulling a turn acceleration a is,

$$\Delta\theta = \frac{\Delta d}{r} \text{ radians,}$$

where

$$\Delta d = \Delta t V \text{ and } r = V^2/a, \text{ or,}$$

$$\Delta\theta = \frac{a}{V} \Delta t.$$

Therefore, the maximum deviation in dive angle is achieved when the ratio a/V is the greatest.

Looking at the maneuverability curves in the flight manual of the A-7A aircraft, the conditions which correspond to the maximum $\Delta\theta$ are a 7g load factor at sea level with a corresponding velocity of 385 knots. These limits are for a 25,000-pound clean aircraft with a TF 30-P-6 engine on a standard day. Based on the above information, it is assumed that 5g is the maximum acceleration that can be pulled by an armed aircraft.

Estimating the maximum dive angle deviation in 0.1 second using 5g for a and 385 knots for V in the expression for $\Delta\Theta$ it is found that

$$\Delta\Theta_{max} = 1.4^{o}$$

Next, the altitude change in one calculation time interval is estimated by assuming that the dive angle at the beginning of this interval is maintained throughout the interval. Inspecting the value of $\Delta\Theta_{max}$, this can be considered a good assumption. Therefore, knowing the aircraft velocity V and dive angle Θ, the altitude change in 0.1 second is,

$$\Delta Y = (0.1) V \sin\Theta$$

Summarizing the results of the estimated maximum deviations in the release conditions in one calculation interval:

a. The deviation in the magnitude of velocity is negligible.
b. The maximum deviation in the dive angle is 1.4 degrees.
c. The change in altitude is $\Delta Y = (0.1)\ V\ \sin \Theta$.

3.6.2 Effects of Maneuvers on Impact Range Calculations

Recall that the objective here is to demonstrate the usability of the impact range from the previous calculation in obtaining the integration step size for the present calculation. This is done by demonstrating that the results of impact range calculation obtained in this manner are, for all practical purposes, the same as those obtained when the exact impact range is used to obtain the integration step size.

The maximum deviation in release conditions due to maneuvers was estimated in the previous subsection. Next, the effects of these deviations on the impact range calculations are presented in two steps: (1) the effect of these release condition deviations on the impact range is estimated to determine how much the impact range might deviate in one calculation interval; (2) the true impact range is perturbed by this amount and is used to obtain the integration interval size.

Maximum impact range deviation in one calculation interval is estimated for a low-drag bomb (M117) and a drogued bomb (Mk 82S) for short, medium, and long trajectories. Results for these cases are given in Table XIV. In obtaining these results, the effects on the impact range due to altitude and dive angle deviations are assumed uncoupled, since the coupling produces only second-order effects. As mentioned previously, only positive-load factors are considered. On the basis of the results in Table XIV, a conservative 5% is assumed across the board as the maximum possible deviation in the impact range due to maneuvers. Note, for long trajectories the maximum deviation is more like 1%.

Finally, Tables XV and XVI show calculation errors in impact ranges for the M117 and Mk 82S bombs using integration steps based on perturbed impact ranges equal to the true impact range minus 5% of this range to account for maneuver deviations. The points in these tables correspond to the points in Tables VII and VIII, which were obtained by using the exact impact range to establish the integration intervals. These results show that even under the most extreme maneuvers the errors in impact-range calculations caused by those maneuvers will be acceptable.

3.7 Advanced Applications

Beyond the application to an aircraft fire control system, use of the algorithm can provide improvements in several areas. Four areas are listed below:

3.7.1 Ballistic Table Calculation

Current standards of accuracy could be maintained, with decreasing computer costs, if the algorithm approach to trajectory integration were substituted for the methods in use. However, as ballistic tables are not calculated often, the cost saving may not justify the cost of developing new computer programs.

Table XIV Deviation in Impact Range Due to Altitude Deviation in 0.1 Second and a 1.4° Deviation in Dive Angle

		RELEASE CONDITIONS			ESTIMATED MAXIMUM RANGE DEVIATION DUE TO:		TOTAL DEVIATION DUE TO MANEUVERS (%)
BOMB	TRAJECTORY LENGTH	ALTITUDE (FEET)	VELOCITY (KNOTS)	DIVE ANGLE (DEGREES)	ALTITUDE DEVIATION (%)	DIVE ANGLE DEVIATION (%)	
LOW DRAG (M117)	SHORT	1,000	200	−50	−1.4	+ 4.4	+ 3.0
	MEDIUM	8,000	400	0	−0.0	+ 2.1	+ 2.1
	LONG	15,000	600	+ 45	0.0	+ 1.0	+ 1.0
HIGH DRAG (Mk 82S)	SHORT	1,000	200	−50	−1.8	+ 4.3	+ 2.5
	MEDIUM	2,500	400	−30	−0.5	+ 2.2	+ 1.7
	LONG	5,000	600	0	0.0	+ 0.9	+ 0.9

NOTES: (1) NEGATIVE DEVIATIONS INDICATE THAT THE IMPACT RANGE BECAME SMALLER

(2) ONLY THOSE DIVE ANGLE DEVIATIONS CAUSED BY POSITIVE LOAD FACTORS WERE CONSIDERED

Table XV M117 Bomb Computation Accuracy with Initial Range Estimate 5% Short

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS) 200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	+1	0.1	0.0	+1	0.1	0.0	0	0.0	0.0
3,000	0	0.0	0.0	+1	0.0	0.0	0	0.0	0.0
5,000	+1	0.1	0.0	+1	0.0	0.0	−1	0.0	0.0
7,000	+1	0.1	0.0	+2	0.1	0.0	−1	0.0	0.0
9,000	+1	0.1	0.0	+1	0.0	0.0	+1	0.0	0.0
11,000	+1	0.1	0.0	+2	0.1	0.0	−3	0.0	0.0
13,000	+1	0.1	0.0	+1	0.0	0.0	−4	0.1	0.0
15,000	+1	0.0	0.0	+2	0.0	0.0	+2	0.0	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 101, JULY 1967

CONDITIONS: LEVEL RELEASE
NO WIND
TARGET AT SEA LEVEL
NO EJECTION VELOCITY

Table XVI Mk 82S Bomb Computation Accuracy with Initial Range Estimate 5% Short

RELEASE ALTITUDE (FT)	RELEASE SPEED (KNOTS)								
	200			400			600		
	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)	RANGE ERROR (FT)	ANGULAR ERROR (MR)	TIME ERROR (%)
1,000	+1	0.3	0.0	+1	0.2	0.0	0	0.0	0.0
2,000	+1	0.3	0.0	0	0.0	0.0	0	0.0	0.0
3,000	0	0.0	0.0	0	0.0	0.0	−7	0.9	0.6
4,000	+1	0.2	0.0	0	0.0	0.3	0	0.0	0.1
5,000	0	0.0	0.0	0	0.0	0.0	−1	0.1	0.0

REFERENCE: NAVAL WEAPONS LABORATORY BALLISTIC TABLE NUMBER 010, OCTOBER 1964

CONDITIONS: 20° DIVE
TARGET AT SEA LEVEL
NO WIND
NO EJECTION VELOCITY

3.7.2 Fire Control System Simulator

One type of work which requires the calculation of many ballistic trajectories is the simulation and evaluation of a fire control system. Many current simulation programs are forced to rely heavily on questionable statistical assumptions about the kind of errors which will be experienced and how they will combine in weapon delivery. The "force" is applied by the sheer cost of running numerous trajectories in a Monte Carlo simulation. By reducing costs, perhaps by a factor of 100 compared to standard ballistic methods, the algorithm can economically provide more accurate and detailed simulation and analysis of proposed fire control systems.

3.7.3 Ballistic Missile Guidance

As the requirements for missile accuracy are tightened, along with a need for greater flexibility, the need for using an explicit guidance technique based on trajectory integration increases.

In some applications, the new integration method will provide the key to obtaining this improved capability without as great an increase in guidance computer power as would be required otherwise.

3.7.4 Command-Guided Bomb

The command-guided bomb is a concept which has been studied as a means of improving the accuracy of conventional weapon delivery without greatly increasing the cost per pound of warhead delivered. The idea is to put a minimal control system and command receiver on an otherwise ordinary bomb. All expensive guidance computers and sensors for target acquisition and bomb tracking are in the aircraft, where their expense may be amortized over many delivered weapons. The bombs are released in much the same way as unguided bombs. Then the guidance system in the aircraft computes commands automatically to make small corrections to the bomb trajectory as the bomb falls. This can yield substantially improved accuracy, compared to unguided bombs, at a cost increase that is small compared to the approaches used in guided missiles.

4.0 AUXILIARY TOPICS

4.1 Computer Requirements

Based on general judgment and automated estimates (Reference 1), it was believed that the algorithm was competitive in terms of computer memory and time requirements. Early in 1968 (before the new atmosphere model was developed), a better estimate was sought. To this end, a basic machine language program was coded. To make the job easier, a simple hypothetical instruction repertoire was assumed. Less than 20 different instructions were used, but the machine was assumed to have an index register and floating point arithmetic. It was assumed for this preliminary estimate that double precision arithmetic was not required. The entire repetitive portion of the algorithm was programmed, including all subroutines. Initial functions were relegated to a separate routine which was neither time critical nor coded.

Table XVII lists the characteristics of the main program (MP) and its eight subroutines. MP uses these subroutines to take N fourth-order, Runge-Kutta integration steps with downrange (X) as the independent variable and one fourth-order, Runge-Kutta rooting step with altitude (Y) as the independent variable. The general layout of the program was chosen to minimize the storage requirement without unreasonably penalizing the execution time. The subroutines are:

Integrate in X (INTX)
Runge-Kutta Equations in X (RKX)
Runge-Kutta Equations in Y (RKY)
Common Runge-Kutta Equations (RKC)
Drag Force Equations (DRAG)
Atmosphere Model (ATMO)
Ballistic Coefficient Lookup (BAL)
Power Subroutine (PWR)

Table XVII Algorithm Code Characteristics

ROUTINE	STORAGE (WORD)	TIMES EXECUTED	NUMBER OF INSTRUCTIONS BY TYPE				
			LOGIC	TRANSFER	ADD	MULTIPLY	DIVIDE
MP	68	1	11	31	11	13	1
INTX	66	N	12	31	12	13	3
RKX	22	4N	2	9	2	7	2
RKY	17	4	2	7	1	4	2
RKC	8	3N + 1	2	1	5	0	0
DRAG	15	4N + 4	5	7	1	2	0
ATMO	91	4N + 4	6	23	4	6	4
BAL	18	4N + 4	6	9	7	1	1
PWR	44	12N + 12	8	9	7	12	1
TOTAL (N=1)	349		375	638	319	430	84
TOTAL (N=5)	349		1,135	1,994	987	1,314	256

MP calls INTX N times for the forward integration, and directly calls DRAG, RKY, and RKS once each for the rooting step. INTX calls DRAG, RKY, and RKS. DRAG calls ATMO and BAL and uses PWR to calculate a square root. ATMO in turns uses PWR to evaluate the exponential function in the atmosphere model. Table XVII shows the resulting number of times each subroutine is used, depending on N, the number of forward integration steps to be taken. At the bottom of the table are the totals for each class of instructions, for one complete program execution, for N = 1 or N = 5. By taking these totals and the typical instruction execution times for a given computer, the approximate execution time can be calculated for that computer if its instruction repertoire does not vary severely from that hypothesized. For example, these instruction times, typical of the IBM 4-Pi Model EC airborne computer, were used:

Logical:	3 microseconds
Data Transfer:	2 microseconds
Floating Point Add:	5 microseconds
Floating Point Multiply:	10 microseconds
Floating Point Divide:	14 microseconds

Using these speeds and the totals in Table XVII, total execution times were calculated:

If N = 1	9.462 milliseconds
If N = 5	29.052 milliseconds

The estimate of 349 word storage does not vary with N. Because of the highly iterative nature of the calculations, a small change in the program can make a large change in the total execution time. In the program as coded, half the total time is used up in the atmosphere calculations. This is an area where many changes are possible, and it has been the subject of considerable research this year. (See Section 4.2.)

More recent, but less detailed, estimates indicate a possible five to one reduction in the computer time required. This is still for the simple but powerful hypothetical computer. The required execution time on the A-7E fire control computer has not been estimated. Two opposing effects will change the estimates:

The computer in the A-7E is slower and less powerful than the hypotheticai computer. This will tend to increase both execution time and memory requirements.

For an operational application, more advantage can be taken of the iterative nature of the calculation to reduce the average execution time. The programming for this will increase the memory requirement, however.

4.2 Derivation of New Atmospheric Model

The available expressions describing atmospheric density are too complex to be programmed efficiently on an airborne computer. There are three expressions, one for each of three altitude regions between sea level and 104,986 feet. It is necessary to evaluate either an exponential function or a function to a nonintegral power in each of these regions. Therefore, a search for a more appropriate representation of the atmospheric density was conducted.

Before proceeding, a brief description of the source of the atmospheric standard is given. Originally, the 1962 U.S. standard atmosphere was used in the calculation of weapon impact ranges. (See Reference 7.) After a more desirable representation of the 1962 atmosphere was developed, it was decided to incorporate the 1965 revision into this model.

In the first 104,986 feet of altitude this revision consists of a slight modification in the expressions and the boundaries between which these expressions apply. The result of these variations is seen as a deviation in about the fifth digit when both expressions are calculated.

The remainder of this Section consists of two parts. Section 4.2.1 includes an outline and the results of the search for a more efficient representation of the atmospheric density. The 1962 U.S. standard atmosphere was used at this time. Section 4.2.2 consists of a more detailed derivation of the series expansion that was finally adopted. In this case the 1965 revision to the U.S. standard atmosphere was included.

4.2.1 Search for an Accurate Power Series

The method used to find an appropriate power series representation of the atmospheric density was to try several expansions and pick the best one. Three power series approximations were obtained by making use of the Chebyshev, Legendre, and Taylor expansions.

The 1962 U.S. standard atmospheric model was used at this point without the 1965 modifications. The atmospheric density equations from this model are given below. These expressions can be used also for comparison with the modified expressions given in Section 4.2.2. Note that when two or more constants appeared in these expressions they were multiplied out. Also, the expression for temperature has been incorporated.

The variables involved in these atmospheric density expressions are:

ρ_w = atmospheric density (lb/ft^3)

H_g = geopotential altitude (ft)

A = geometric altitude (ft)

where,

$$H_g = \frac{A\,(20{,}855{,}531)}{A + 20{,}855{,}531} \qquad (4\text{-}1)$$

First Region: $0 \leqslant H_g < 36,089.239$

$$\rho_w = \frac{22.03627125}{[288.15 - (.0019812)(H_g)]}[1 - (6.8755856X10^{-6})(H_g)]^{5.2558761} \qquad (4\text{-}2)$$

Second Region: $36,089.239 \leqslant H_g < 65,616.798$

$$\rho_w = (.02271887182)e^{-4.8063428X10^{-5}(H-36,089.239)} \qquad (4\text{-}3)$$

Third Region: $65,616.798 \leqslant H_g < 100,000$

$$\rho_w = \frac{1.190681437\,[1 + 1.4068775 \text{ X } 10^{-6}(H - 65616.798)]^{-34.163195}}{[216.65 + .0003048\,(H_g - 65616.798)]} \qquad (4\text{-}4)$$

Note that the upper limit of 100,000 feet was chosen rather arbitrarily. The actual value given for this limit was 104,986.877 feet.

The procedure for finding a power series represenation using the Chebyshev expansion will be outlined next. In principle this procedure is the same as that used for the Legendre expansion. A detailed description of the Legendre expansion is included in Section 4.2.2; therefore, the Legendre expansion will not be discussed here. Since the Taylor series expansion is fairly well known, it will not be discussed either.

In preparation for the Chebyshev expansion, the atmospheric density expressions in each altitude region need to be transformed so that the range of the independent variable x is, $-1 \leqslant x \leqslant 1$. Then, the object is to expand ρ_w, the atmospheric density, in terms of the Chebyshev polynomials of the second kind.

These polynomials are given below:

$$\begin{aligned}
U_0(x) &= 1 \\
U_1(x) &= 2x \\
U_2(x) &= 4x^2 - 1 \\
U_3(x) &= 8x^3 - 4x \qquad (4\text{-}5) \\
U_4(x) &= 16x^4 - 12x^2 + 1 \\
U_5(x) &= 32x^5 - 32x^3 + 6x
\end{aligned}$$

Etc.

These functions are orthogonal with respect to the weight function $\sqrt{1 - x^2}$.

The orthogonality conditions are:

$$\int_{-1}^{1} \sqrt{1 - x^2}\, U_n(x)\, U_m(x)\, dx = \pi/2 \quad , \quad m = n$$
$$= 0 \quad , \quad m \neq n \tag{4-6}$$

The expansion in terms of the Chebyshev polynomials is of the form

$$\rho_w = AC(1)\ U_0(x) + AC(2)\ U_1(x) + AC(3)\ U_2(x) \ \ldots.. \tag{4-7}$$

where the AC's are constants found by making use of the orthogonality conditions. For example, to find AC(1), multiply (4-7) through by $U_0 \sqrt{1 - x^2}$ and integrate over the region (-1, 1). Because of the orthogonality conditions and the fact that the AC's are constants, the result is:

$$\int_{-1}^{1} \rho_w(x) \sqrt{1 - x^2}\ U_0(x)\ dx = AC(1)\, \pi/2 \tag{4-8}$$

Therefore,

$$AC(1) = \frac{2}{\pi} \int_{-1}^{1} \rho_w(x) \sqrt{1 - x^2}\ U_0(x)\ dx \tag{4-9}$$

Similarly,

$$\left.\begin{aligned} AC(2) &= \frac{2}{\pi} \int_{-1}^{1} \rho_w(x) \sqrt{1 - x^2}\ U_1(x)\ dx \\ AC(n) &= \frac{2}{\pi} \int_{1}^{1} \rho_w(x) \sqrt{1 - x^2}\ U_{n-1}(x)\ dx \end{aligned}\right\} \tag{4-10}$$

Having found as many AC's as desired, the expression for ρ_w in (4-7) is known completely. Since the U's in this expression are all polynomials in x, one can collect like powers of x to get ρ_w as a power series in x. After transforming these expressions so that the altitude A becomes the independent variable, one obtains the desired power series expressions for ρ_w in terms of the altitude A.

The resulting coefficients of the different powers of the geometric altitude, A, obtained from the different expansions, are given in Table XVIII. In each case, six terms were used in the expansion. Table XIX compares the performance for the three different expansions. Note how much better the series resulting from the Legendre expansion is than the other two series.

Table XVIII Coefficients Of The Different Powers Of A In The Expansion Of ρ

		COEFFICIENTS C_N FOR THE EXPRESSION $\rho = \sum C_N A^N$		
ALTITUDE REGION (FT)	N	COEFFICIENTS FROM THE CHEBYSHEV EXPANSION	COEFFICIENTS FROM THE LEGENDRE EXPANSION	COEFFICIENTS FROM THE TAYLOR EXPANSION
A FROM ZERO TO 36,151.797	0	$7.687393941 \times 10^{-2}$	$7.647500017 \times 10^{-2}$	$7.647500000 \times 10^{-2}$
	1	$2.407296101 \times 10^{-6}$	$-2.237784147 \times 10^{-6}$	$-2.237783949 \times 10^{-6}$
	2	$4.815919194 \times 10^{-11}$	$2.515493058 \times 10^{-11}$	$5.030975273 \times 10^{-11}$
	3	$1.474875550 \times 10^{-15}$	$-1.319129466 \times 10^{-16}$	$-9.157272234 \times 10^{-16}$
	4	$3.544103811 \times 10^{-20}$	$2.986533408 \times 10^{-22}$	$5.034864967 \times 10^{-20}$
	5	$-3.383920382 \times 10^{-25}$	$-1.606578496 \times 10^{-28}$	$-5.983560285 \times 10^{-26}$
A FROM 36,151.797 TO 65,823.897	0	$2.358894826 \times 10^{-1}$	$1.242282588 \times 10^{-1}$	$*2.271887183 \times 10^{-2}$
	1	$-1.649391491 \times 10^{-5}$	$-5.593402375 \times 10^{-6}$	$-1.088171034 \times 10^{-6}$
	2	$5.375930550 \times 10^{-10}$	$1.156099115 \times 10^{-10}$	$5.222455116 \times 10^{-11}$
	3	$-9.448623261 \times 10^{-15}$	$-1.349502937 \times 10^{-15}$	$-2.511405471 \times 10^{-15}$
	4	$8.578708979 \times 10^{-20}$	$8.702175740 \times 10^{-21}$	$1.210099127 \times 10^{-19}$
	5	$-3.153672168 \times 10^{-25}$	$-2.423923389 \times 10^{-26}$	$-5.842308620 \times 10^{-24}$
A FROM 65,823.897 TO 100,000	0	$2.660965125 \times 10^{-1}$	$1.169028307 \times 10^{-1}$	NO DATA
	1	$-1.360722778 \times 10^{-5}$	$-4.705422820 \times 10^{-6}$	
	2	$2.928061740 \times 10^{-10}$	$8.125386209 \times 10^{-11}$	
	3	$-3.244227337 \times 10^{-15}$	$-7.409812969 \times 10^{-16}$	
	4	$1.827244849 \times 10^{-20}$	$3.523424824 \times 10^{-21}$	
	5	$-4.153857778 \times 10^{-26}$	$-6.919278720 \times 10^{-27}$	

*IN THIS REGION THE TAYLOR EXPANSION IS ABOUT A = 36,151.797 INSTEAD OF A = 0 AS IN OTHER CASES

Table XIX Comparing Values of ρ_W Calculated by the Different Expansions to These Calculated by the Adopted Standard

ALTITUDE REGION (FT)	THE SMALLEST NUMBER OF DIGITS IN AGREEMENT TO WITHIN ONE HALF IN THE LAST DIGIT		
	CHEBYSHEV EXPANSION	LEGENDRE EXPANSION	TAYLOR EXPANSION
ZERO TO 36,151.797	3	9	0
36,151.797 TO 65,823.897	3	6	0
65,823.897 TO 100,000	3	5	NO DATA

4.2.2 Derivation of Power Series from Legendre Expansion

The series derived from the Legendre expansion was adopted since it produced by far the most accurate representation of the atmospheric density. A detailed description of the derivation of this series was not given in the previous section and is given here with the 1965 modifications to the standard atmosphere. Section 4.2.2.1 includes the 1962 U.S. standard atmospheric density with 1965 modifications. Transformations necessary to make air density expressions compatible with the Legendre polynomial are given in Section 4.2.2.2. The Legendre expansion is described in Section 4.2.2.3, and the results are given in Section 4.2.2.4.

4.2.2.1 Expressions for Atmospheric Density. The expressions for the atmospheric density (in lb/ft^3), including the 1965 modifications, are given below. First, let,

$$H_g = \text{geopotential altitude (ft)}$$
$$A = \text{geometric altitude (ft)} \tag{4-11}$$

where the relationship between A and H is,

$$H_g = \frac{A(20{,}855{,}531)}{A + 20{,}855{,}531} \tag{4-12}$$

First Region, $0 \leq H_g < 36{,}089$

$$\rho_w = \frac{22.03644887}{[288.15-(.0019812)(H_g)]} [1 -(6.8755856X10^{-6})(H_g)]^{5.2558761} \tag{4-13}$$

Second Region, $36{,}089 \leq H_g < 65{,}617$

$$\rho_w = (0.227190667)\ e^{-4.80634274X10^{-5}(H_g - 36{,}089)} \tag{4-14}$$

Third Region, $65{,}617 \leq H_g \leq 104{,}987$

$$\rho_w = \frac{(1.190694356)[1 +(1.40687745X10^{-6})(H_g - 65{,}617)]^{-34.163195}}{[216.65 +(0.0003048)(H_g - 65{,}618)]} \tag{4-15}$$

4.2.2.2 Transformation of Variables. The input to the weapon control system, in practice, will be the altitude A, not H_g. Therefore, the independent variable H_g in the expressions for the atmospheric density given above will be transformed to A by the use of (4-12). Also, in each region the expansion will be about the lower boundary limit of the region.

The Legendre polynomials used are orthogonal in the region $-1 \leqslant x \leqslant 1$, where x is the independent variable for these functions. Therefore, it is necessary to make a linear transformation of variables in the expressions for ρ_w in each region so ρ_w becomes a function of x, where, $-1 \leqslant x \leqslant 1$. After the expansion in terms of the Legendre polynomials is completed, one can transform back to the variable A. The appropriate transformation for each region is

$$x = \left[\frac{2}{A_{max} - A_{min}}\right] \left[A - A_{min}\right] - 1 \qquad (4\text{-}16)$$

where A_{max} and A_{min} denote the upper and l wer boundary limits, respectively, of A in each region.

4.2.2.3 Legendre Expansion. Different numbers of terms were tried for representing the atmosphere in the first region. It was decided to use five terms or a fourth-order representation. In the next two regions six terms or fifth-order representations were used.

Next, the object is to expand $\rho_w(x)$, $-1 \leq x \leq 1$, in terms of the Legendre polynomials which are given below.

$$\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= \frac{3}{2}x^2 - \frac{1}{2} \\
P_3(x) &= \frac{5}{2}x^3 - \frac{3}{2}x \\
P_4(x) &= \frac{35}{8}x^4 - \frac{15}{4}x^2 + \frac{3}{8} \\
P_5(x) &= \frac{63}{8}x^5 - \frac{35}{4}x^3 + \frac{15}{8}x
\end{aligned} \qquad (4\text{-}17)$$

Etc.

The orthogonality conditions are:

$$\int_{-1}^{1} P_n(x)\ P_m(x)\ dx = 0 \quad , \quad m \neq n$$
$$= \frac{2}{2m+1} \quad , \quad m = n \tag{4-18}$$

The expansion in terms of the Legendre functions is of the form:

$$\rho_w = DC(1)\ P_0(x) + DC(2)\ P_1(x) + DC(3)\ P_2(x) + \ldots \tag{4-19}$$

where the DC's are constants that are found by making use of the orthogonality conditions. For example, to find DC(1), multiple (4-19) through by $P_0(x)$ and integrate over the region (-1,1). The result is:

$$\int_{-1}^{1} \rho_w(x)\ P_0(x)\ dx = 2DC(1) \tag{4-20}$$

Therefore,

$$DC(1) = \frac{1}{2} \int_{-1}^{1} \rho_w(x)\ P_0(x)\ dx \tag{4-21}$$

Similarly,

$$DC(2) = \frac{3}{2} \int_{-1}^{1} \rho_w(x)\ P_1(x)\ dx$$
$$DC(n+1) = \frac{2n+1}{2} \int_{-1}^{1} \rho_w(x)\ P_n(x)\ dx \tag{4-22}$$

As mentioned before, five terms are used in the expansion for the first region, and six in the other two regions. The coefficients, DC(1) through DC(5) or DC(6), are found by using (4-22). Next, the object is to expand ρ_w in powers of $(A-A_{min})$. This is done in two steps. First, the result of substituting (4-17) in (4-19) and collecting like powers of X is;

$$\rho_w = [DC(1) - 1/2\,DC(3) + \frac{3}{8}DC(5)] + x\,[DC(2) - \frac{3}{2}DC(4) + \frac{15}{8}DC(6)]$$
$$+ x^2\,[\frac{3}{2}DC(3) - \frac{15}{4}DC(5)] - x^3\,[\frac{5}{2}DC(4) - \frac{35}{4}DC(6)] \qquad (4\text{-}23)$$
$$+ x^4\,[\frac{35}{8}DC(5)] + x^5\,[\frac{63}{8}DC(6)],$$

or,

$$\rho_w = EC(1) + EC(2)\,x + EC(3)\,x^2 + EC(4)\,x^3 + EC(5)\,x^4 + EC(6)\,x^5, \qquad (4\text{-}24)$$

respectively. The last term in the above expression does not exist in the expression for the first region.

Next, it is desirable to get ρ_w as a function of $(A - A_{min})$, where A_{min} is the lower limit in each region. The relationship between x and $(A - A_{min})$ is

$$x = S\,(A - A_{min}) - 1 \qquad (4\text{-}25)$$

where $S = 2/(A_{max} - A_{min})$. (See (4-16).) The result of substituting (4-25) into (4-24) and collecting like powers of $(A - A_{min})$ is,

$$\rho_w = [EC(1) - EC(2) + EC(3) - EC(4) + EC(5) - EC(6)]$$
$$+ (A - A_{min})\,[EC(1) - 2EC(3) + 3EC(4) - 4EC(5) + 5EC(6)]\,S$$
$$+ (A - A_{min})^2\,[EC(3) - 3EC(4) + 6EC(5) - 10EC(6)]\,S^2$$
$$+ (A - A_{min})^3\,[EC(4) - 4EC(5) + 10EC(6)]\,S^3 \qquad (4\text{-}26)$$
$$+ (A - A_{min})^4\,[EC(5) - 5EC(6)]\,S^4$$
$$+ (A - A_{min})^5\,[EC(6)]\,S^5$$

or

$$\rho_w = LC(1) + LC(2)\ (A - A_{min}) + LC(3)\ (A - A_{min})^2 + LC(4)\ (A - A_{min})^3$$

$$+ \quad LC(5)\ (A - A_{min})^4 + LC(6)\ (A - A_{min})^5 \quad , \ldots\ldots \qquad (4\text{-}27)$$

respectively. Note that (4-26) and (4-27) represent the six-term expansion. To obtain the five-term expansion simply set EC(6) and, consequently, LC(6) to zero.

The LC's are the desired coefficients. A computer program has been written to calculate these LC's.

4.2.2.4 Results. The coefficients of $(A - A_{min})$ for the calculation of the atmospheric density and atmospheric density ratio are give. .n Table XX. Table XXI describes the accuracy of the power series representation of the atmospheric density expressions. This accuracy is described by giving the smallest number of digits in agreement between the developed power series and the expressions approximated.

When using the CDC 6600 computer, this series can be evaluated approximately 25 times faster than the original expressions. As seen in Table XXI, the agreement between this series and the expressions being approximated is at least seven digits for altitudes below 37,000 feet and five digits for altitudes below 100,000 feet.

Table XX Coefficients of (A – A min) in the Expressions for Atmospheric Density and Density Ratio

ALTITUDE REGION (FT)	N	COEFFICIENTS OF $(A - A_{min})^N$ FOR ATMOSPHERIC DENSITY (LB/CU FT)	COEFFICIENT OF $(A - A_{min})^N$ FOR ATMOSPHERIC DENSITY RATIO
0 TO 36,151.558	0	7.6475577×10^{-2}	9.9999949×10^{-1}
	1	$-2.2377695 \times 10^{-6}$	$-2.9261216 \times 10^{-5}$
	2	$2.5148808 \times 10^{-11}$	$3.2884740 \times 10^{-10}$
	3	$-1.3144741 \times 10^{-16}$	$-1.7188146 \times 10^{-15}$
	4	$2.8413555 \times 10^{-22}$	$3.7153745 \times 10^{-21}$
36,151.558 TO 65,824.100	0	2.2718915×10^{-2}	2.9707396×10^{-1}
	1	$-1.0879609 \times 10^{-6}$	$-1.4226246 \times 10^{-5}$
	2	$2.6036523 \times 10^{-11}$	$3.4045523 \times 10^{-10}$
	3	$-4.0790415 \times 10^{-16}$	$-5.3337805 \times 10^{-15}$
	4	$4.3207373 \times 10^{-21}$	$5.6498234 \times 10^{-20}$
	5	$-2.4239199 \times 10^{-26}$	$-3.1695330 \times 10^{-25}$
65,824.100 TO 105,518.180	0	5.4956781×10^{-3}	7.1861834×10^{-2}
	1	$-2.6989696 \times 10^{-7}$	$-3.5291898 \times 10^{-6}$
	2	$6.7688823 \times 10^{-12}$	$8.8510333 \times 10^{-11}$
	3	$-1.1099512 \times 10^{-16}$	$-1.4513792 \times 10^{-15}$
	4	$1.1761356 \times 10^{-21}$	$1.5379224 \times 10^{-20}$
	5	$-6.0891352 \times 10^{-27}$	$-7.9621917 \times 10^{-26}$

NOTE: THE SYMBOL A DENOTES GEOMETRIC ALTITUDE AND A_{min} DENOTES THE LOWER ALTITUDE LIMIT FOR EACH REGION.

Table XXI Comparison of the Power Series to the Expressions They Represent

	ALTITUDE REGION (FT)	COMPARISON *	ORDER OF SERIES
FIRST REGION	0 TO 36,151.6	7	4
SECOND REGION	36,151.6 TO 65,824.1	5	5
THIRD REGION	65,824.1 TO 100,000	5	5
	100,000 TO 105,518.2	4	

*NOTE THIS COLUMN GIVES THE SMALLEST NUMBER OF DIGITS IN AGREEMENT TO WITHIN ONE HALF IN THE LAST DIGIT

5.0 CONCLUSIONS

The technique described in this document for calculating weapon impact points applies to any unpowered, unguided projectile. It also applies to some powered and/or guided projectiles which have a substantial ballistic trajectory segment. The reason for this wide applicability is that an accurate representation of the differential equations of motion is integrated directly. The efficiency of the new algorithm combined with the high speed of the new computers permits both accuracy and flexibility in airborne fire control calculation.

It has been established that the algorithm can predict the trajectories of many types of projectiles. The primary application, of course, is in airborne fire control systems. It can be used in weapon release calculations for most air-to-ground weapons and some air-to-air weapons. The algorithm could also be used in direction of ground-to-ground or ground-to-air guns, unguided rockets, or guidance calculations for guided weapons which operate in a semi-ballistic mode.

There are sound technical reasons to believe that results equal to those shown in this document can be produced under similar conditions for any weapon with similar characteristics.

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